UNSATURATED MODELLING OF DUAL SCALE FLOW IN LIQUID COMPOSITE MOULDING PROCESSES

Y. Wang and S.M. Grove

Advanced Composites Manufacturing Centre
School of Engineering, University of Plymouth, Plymouth, UK, PL4 8AA

ABSTRACT: An unsaturated flow model is incorporated into a dual-scale model previously developed for resin flow through woven fibre mats. In this model, the two different scale flows are simulated at a same numerical scale. A sink and a source term are introduced for the two different scaled flows, respectively, based on mass conservation in the dual-scale domain. With this model, the micro flow inside fibre tows can be described in detail, including the axial flow in flow direction and the transverse flow in the cross section of fibre tows, while capillary effects have been included in the axial flow within the fibre tow. The transverse flow is simply regarded as being caused by the saturation difference between the inter-tow gaps and the intra-tow pore space. To close the equations in the model, a resin retention characteristic and microscopic diffusivity have been assumed (in the absence of experimental data). Finite element discretization of the governing equation is defined, and a simple implicit time integration method is suggested. A 1D example of RTM has been modelled, and results discussed.

KEYWORDS: RTM, Dual-scale model, Unsaturated flow, Capillary effect, FEA

INTRODUCTION

Liquid Composite Moulding (LCM), in its various industrial forms, is a widely used technology in the manufacture of advanced composite components. Because of its efficiency and cost saving, mathematical modelling of LCM processes has attracted more and more research interest in recent decades, and is playing an increasingly important role in technology applications. Considerable research effort has been devoted to process modelling, and several numerical simulation procedures have been developed [1,2]. In early studies, resin flow through dry fabric beds was modelled as a single flow at a single scale. The fibre preform is assumed to be fully saturated behind the resin front, and therefore Darcy’s law is adopted to describe the superficial velocity of the resin flow, in which the pressure gradient in resin saturated region is regarded as the driving force. Incorporating the mass conservation condition, Darcy’s law can be used to determine the progress of the resin font [3,4] where, normally, the permeability of porous fabrics is assumed to be a constant in a representative elementary volume (REV). However, the ‘fully saturated’ assumption is not a reality, especially for woven fibre mats, which consist of discrete fibre tows, and are microscopically heterogeneous. Investigation has shown that two different flow rates (a fast macro-flow in the gaps between fibre tows and a slow micro-flow in the passages inside tows) coexist in resin infusion process for this type of preform. Voids may be retained in tows after the resin has passed, due to incomplete impregnation.

An understanding of the formation of voids and/or dry spots is clearly important, as these features can have serious implications for composite quality and performance. This has been investigated through two-scale or two-layer models of flow through woven fibre mats. Lekakou and Bader [5] proposed a model of the macro- and micro-infiltration using Darcy's law, where separate permeabilities were used for different scale flows and capillary pressures were considered. Recently, Pillai [1] developed a rigorous dual-scale mathematical model which simulates the macro-flow using a volume average method. A sink term of the macro-flow is introduced which couples the two different scaled flows together based on a mass conservation. Darcy's law has been proven to be valid for the macro-flow, while an idealized cylindrical microscopic impregnation of fibre tows was assumed.

In this paper, based on the dual-scale model, we try to simultaneously model the macro- and micro-infiltration in LCM process in a same numerical scale. Similar to the sink term in the macro-flow case, a source term is introduced to the micro-domain which is caused by the resin sink from the identical local macro-domain. The resin macro-flow is assumed to have a distinct resin front, but in the micro-domain, no distinct resin front exists because of source term. The approach here is to adopt the variation of saturation to describe the resin impregnation inside fibre tow regions. A 1D numerical simulation example using this model will be presented.

THEORY

2.1 Governing equation

Based on the consideration of the dual-scale nature of woven fabrics, Pillai, employing the volume average methods, developed the following governing equations for the resin macroscopic flow in inter-tow gaps [1]:

$$\nabla \cdot \langle \mathbf{v}_g \rangle = -S \tag{1}$$

$$\langle \mathbf{v}_g \rangle = -\frac{K_g}{\mu} \nabla \langle P_g \rangle^g \tag{2}$$

where, $\langle \mathbf{v}_g \rangle$ is volume average resin flow velocity, S is a sink term accounting for the rate of resin absorption by the tows in per unit volume, $\langle P_g \rangle^g$ is gap average modified resin pressure, which is defined as $P_g = p_g + \rho_g g h$, μ is resin viscosity, K_g is resin permeability in inter-tow gaps. Theoretically, S is the mass loss in gaps due to resin infusion into fibre tows from the gaps, which can be expressed as:

$$S = \frac{1}{V} \int_{S_{gt}} \mathbf{v}_g \cdot \mathbf{n}_{gt} \, ds \tag{3}$$

where S_{gt} is total area of the tow-gap interface and $\mathbf{n}_{gt}$ is the normal vector of the surface.

Eqs. 1 and 2 assume that a distinct resin front exists in inter-tow gaps, which separates the space of the gaps into a fully impregnated region and a dry region, and they are only applicable for the part of resin flowing in the saturated region. However, the resin flowing inside fibre tows normally has a slow velocity because of high resistance there. The resin movement inside fibre tows due to the local pressure gradient will lag behind that in inter-tow gaps, and resin will sink from inter-tow gaps into fibre tows at front region. As a result, an unsaturated region exists at the resin front in fibre tows, where the resin content is variable. In view of the fact that it is difficult to outline a distinct sharp resin front at numerical scale for the resin flow inside fibre tows, we believe that using saturation to describe the flow in fibre tow is a reasonably good solution [6,7]. By introducing the saturation, the resin flow inside the fibre tows can be expressed as:

$$\frac{\partial}{\partial t}\left(\varepsilon_f \rho \theta_f\right) = -\nabla\left(\rho\theta_f \left\langle \mathbf{v}_f \right\rangle\right) + R_f \tag{4}$$

$$\left\langle \mathbf{v}_f \right\rangle = -\frac{K_f}{\mu}\nabla\left\langle P_f \right\rangle^f \tag{5}$$

Eq. (4) is mass conservation equation, where θ_f is the saturation, which is the fraction of the void in fibre tows been occupied by resin, $\varepsilon_f = V_f / V$ is the fibre void fraction, V_f is the total volume of voids in fibre tows, ρ is resin density, R_f is a source term, a homogeneous mass production rate from other generating phase per unit volume. According to mass conservation, the source term in fibre tows is equal to the sink term in Eq. (1), i.e.

$$R_f = \rho S \tag{6}$$

Eq. (5) is the Darcy law applied on the flow inside fibre tows, where $\left\langle \mathbf{v}_f \right\rangle$ is the average superficial velocity of resin inside tow, K_f is permeability inside fibre tows. $\left\langle P_f \right\rangle^f$ is an average modified resin pressure inside fibre tow, which consists of the following components:

$$\left\langle P_f \right\rangle^f = \left\langle P_g \right\rangle^g + \left\langle P_f \right\rangle_f^f + \left\langle P_f \right\rangle_c^f \tag{7}$$

where $\left\langle P_g \right\rangle^g$ is the resin pressure in gaps as defined before, $\left\langle P_f \right\rangle_f^f$ is the resin pressure insider tows caused by the fibre deformation, which will not be considered in this paper. $\left\langle P_f \right\rangle_c^f$ is the resin pressure inside tows caused by the capillary effect.

Equations (1-7) fully describe the resin flow in a dual-scale fabric mat. But using Eq. 3 to calculate S is not easy and we need to know the detail of the woven structure of fabric reinforcement so that different cross sections of different architectures can be described in detail [8]. As a simpler alternative, we regard the saturation difference as the driving force for the sink/source term, assuming that once the fibre tow is fully saturated the local sink/source process will stop. Because the sink is a flow crossing the surface of fibre, similar to the treatment by Pillai [1], we also idealize that the cross section of fibre tows is a circle, and ignore the dynamic distribution of resin inside fibre tows. Finally the sink/source term can be approximated as:

$$R_f = \rho S = \rho D(\frac{\theta_g - \theta_f}{r_{tow}^2}) \tag{8}$$

where D is a parameter called as diffusivity (m^2/s) of the fibre tows, θ_g is the saturation at gap-tow interface, which is assumed to be 1 when the gap is occupied and 0 otherwise. θ_f is the saturation in fibre tow and r_{tow} is a normalized fibre radius which can be assumed as an average value for a deformed fibre tow.

2.2 The characteristics of dual-scale fibre reinforcement

To close the above equations, we need to know the permeability K_g and K_f, capillary component pressure $\langle P_f \rangle_c^f$ and diffusivity D. The Carman-Kozeny equation is normally employed to evaluate the permeability for axial flow along fibres and transverse flow across fibres when the fibres are considered as solid and of circular cross-section [5]:

$$K = \frac{r_f^2}{4k} \frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{9}$$

where r_f is radius of fibre tow for macro-flow and is fibre radius for micro-flow. k is the Kozeny constant and ε is porosity as defined before (ε_g for macro-flow and ε_f for micro-flow).

It must be noticed that Eq. (9) is suggested for saturated flow but not strictly suitable for the unsaturated flow defined by Eqs. (4) and (5). In this paper, Eq. (9) was employed to estimate the macro-scale permeability K_g, but for the micro-scale permeability, which depends on the saturation of voids, we employed an empirical model by Gilham et al. [9] from water-soil research:

$$K_f = a\theta^n \tag{10}$$

where θ is saturation of fibre tows, a and n are two empirical parameters.

For the capillary component pressure, a model proposed for water retention characteristic [10] was adopted, which assumes that capillary pressure varies with saturation due to the change of the size of occupied pore space:

$$\langle P_f \rangle_c^f = \phi_0 + P_0 \left(\exp(\alpha\theta) - \exp(\beta(1-\theta))\right) \tag{11}$$

where ϕ_0, P_0, α and β are four constants.

There is no reference data available for the value of diffusivity for fibre fabrics. According to its definition, diffusivity is related to permeability by [10]:

$$D(\theta) = \frac{K_f(\theta)}{\mu} \frac{\partial \langle P_f \rangle_c^f}{\partial \theta} \tag{12}$$

Using Eqs. (10) and (11), Eq. (12) can be rewritten as:

$$D = \frac{a\theta^n}{\mu} P_0\left(\alpha \exp(\alpha\theta) + \beta \exp(\beta(1-\theta))\right) \qquad (13)$$

In the next section, the proposed model will be applied to simulate a RTM process using finite element methods.

EXAMPLES

An example of 1D flow in a rectangular domain with resin being injected at one end (Fig.1) was employed to test the model. Due to lack of data for the capillary pressure inside fibre tows, an experimental result provided by Lin et al. [11] was adopted for illustration. Fig. 2 shows the fitting result of Eq. (11) to the experimental data. The corresponding fitting data are listed in Table 1. The other dual-scale parameters are listed in Table 2.

Table 1. Resin retention characteristic parameters

ϕ_0 (Pa)	P_0 (Pa)	α	β
–2009	2.823×10^{-3}	13.57	15.57

Table 2. Basic parameters used in simulation

ε_g	ε_f	μ_{resin} Pa.s	r_{tow} mm	K_g mm^2	a mm^2	n
0.1	0.3	0.4	0.3	5.6×10^{-5}	9.8×10^{-7}	2.0

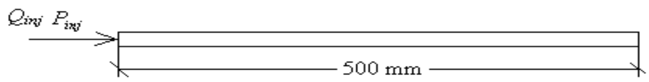

Fig. 1. 1D flow example

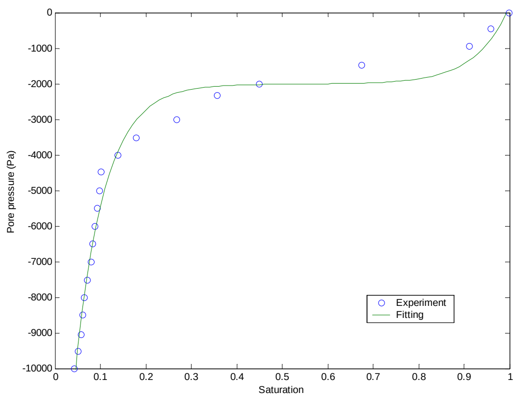

Fig. 2. Capillary pressure with saturation (data from Lin et al., 1998)

Figs. 3 to 6 show the modelling results at two different time steps under injection pressure P_{inj} = 150 kPa. We can see that a stable numerical result has been obtained. Fig. 3 shows the resin profile in the inter-tow gaps with an obvious sharp resin front. Two distinct regions in the gaps (fully resin impregnated or completely dry), separated by the sharp resin front, have been modelled successfully. Fig. 4 shows the corresponding resin pressure profiles in the gaps. It can be seen that the pressure distributions are concave curves rather than linear. This is consistent with the analysis, since, according to Equation (16), the pressure should have a concave shape when S > 0. Figs. 5 and 6 show the corresponding resin saturation and pressure profiles in fibre tows. The resin saturation profiles do not have a significant sharp front but exhibit three distinct regions: (i) the resin front lags behind that in the gaps; (ii) a saturated region expands starting from the resin inlet port; (iii) an unsaturated region having a variation in resin content exists in the middle. The existence of three distinct regions is consistent with the experimental observation [12]. It has been noticed that the middle unsaturated part expands faster than the saturated part behind it. This implies that that the pressure-driven resin flow in the fibre tows is slower than the sink from the gaps because of the very low permeability in fibre tows. Thus it is concluded that resin infiltration from the macro-scale to the micro-scale plays an important role in the impregnation of fibre tows.

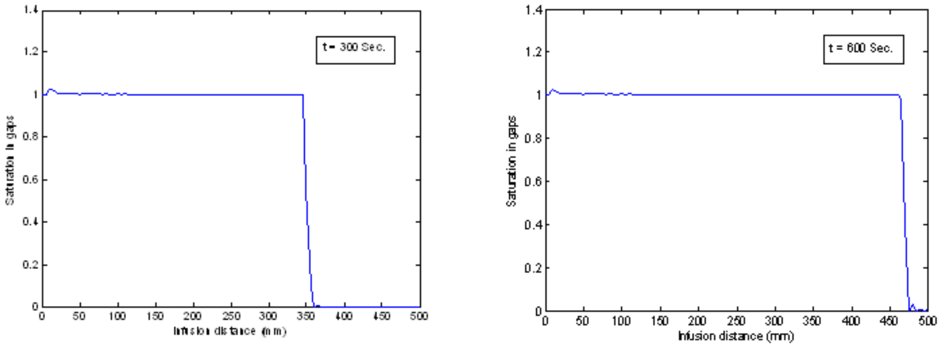

Fig. 3. Dual-scale simulation: saturation profiles in gaps

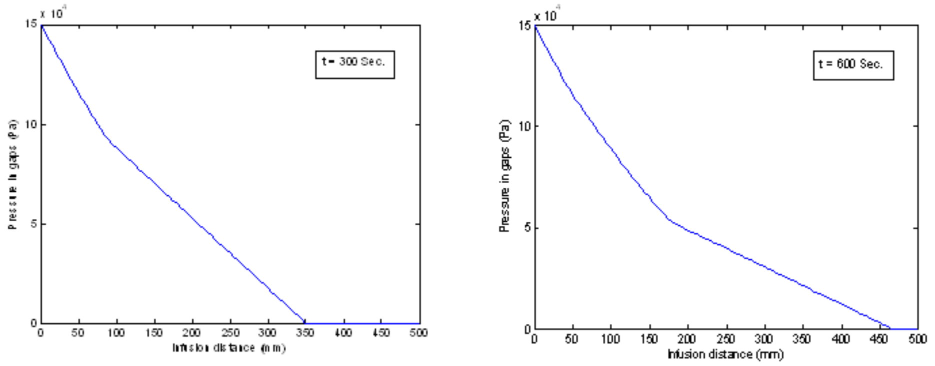

Fig. 4. Dual-scale simulation: pressure profiles in gaps

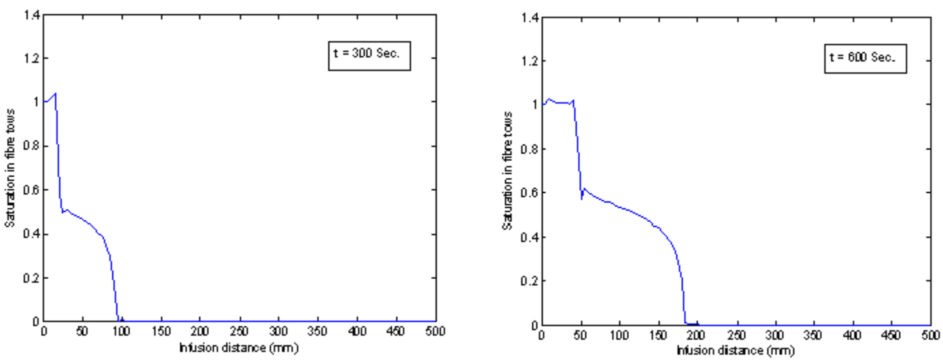

Fig. 5. Dual-scale simulation: saturation profiles in fibre tows

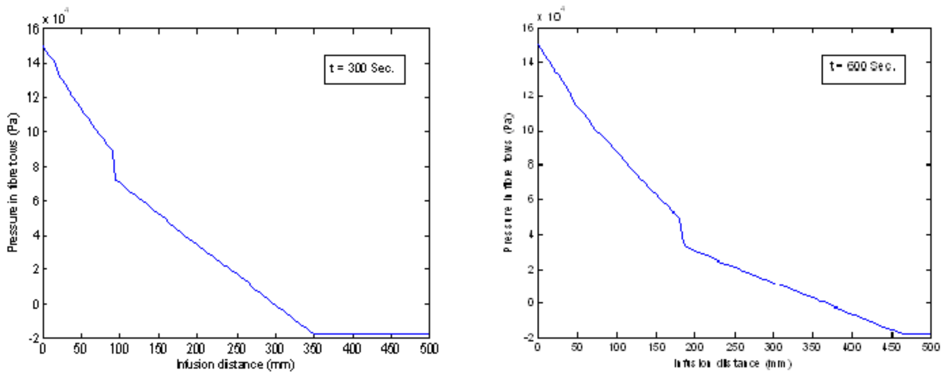

Fig. 6. Dual-scale simulation: pressure profiles in fibre tows

CONCLUSIONS

A dual-scale model for resin flow through woven fibre mats has been proposed by previous researchers. In this paper, this model has been successfully combined with an unsaturated flow model used in other disciplines. In the new model, the two different scale flows are simulated at a same numerical scale. A sink and a source term are introduced for the two different scaled flows, respectively, based on the mass conservation in the dual-scale domain. The results show that the model works very well. Stable numerical results have been obtained. Distinct resin sharp front has been simulated for the macro-scale flow in the inter-tow gaps, while smooth variation of the resin saturation in fibre tows has been modelled. The result of the smooth variation of the micro-scale resin saturation can be used to explain the bubble formation behind the sharp resin front. It is hoped that this theoretical approach will be applicable to future studies of void formation and transport.

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