NONCONFORMING ELEMENTS FOR LIQUID COMPOSITE MOLDING PROCESS SIMULATIONS

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ABSTRACT: Liquid Composite Molding (LCM) processes are now a prevalent group of manufacturing methods for advanced composite materials. They offer many advantages over more traditional manufacturing methods, such as the ability to deal with large and complex shapes. Numerical simulations can lead to better predictions of process parameters. The standard procedure for the simulation of these processes is to use a Control Volume (CV) method. One problem with the CV method is that resin mass is not conserved on an element level, and this has consequences for accuracy. An attractive alternative to the CV method is to use a *single* grid of non-conforming finite elements. Such non-conforming elements encompass essential mass conservation properties. In this study it is shown how the standard non-conforming triangular element can be adjusted to ensure mass conservation on the element level and to ensure continuity of the fluid flux across inter-element boundaries. Numerical experiments are carried out which show that single grids of such elements, and nonconforming quadrilateral elements, produce accurate results in the case of the Injection Compression Molding process.

KEYWORDS: Simulation, Finite Element Method, Control Volumes, Injection/Compression Molding, Conservation of mass, Nonconforming elements

INTRODUCTION

Liquid Composite Molding (LCM) is a family of advanced composite materials manufacturing processes, including Resin Transfer Molding (RTM), Injection Compression Molding (I/CM) and Vacuum Assisted Resin Transfer Molding (VARTM). In these processes, a fibrous material is laid out in a mould, compacted under pressure, impregnated with a polymer resin and finally allowed to cure. In RTM, rigid molds are used to compact the fibrous material to its final thickness before resin injection. In I/CM, the upper mold is brought down with velocity- or force-control but not to the part's final thickness; this allows for ease of resin flow during injection and the final compaction to the final thickness helps drive the injected fluid through the part. In VARTM, a flexible bag covers one side of the part and vacuum pressure drives the fluid through the fibrous material. The LCM processes offer many advantages over more traditional manufacturing methods, such as the ability to deal with large and complex shapes and the reduction in exposure to harmful emissions.

Mathematical models and numerical simulations of the LCM manufacturing processes can lead to better predictions of flow paths, mould filling times, required mould forces, preform final thicknesses and of the optimal positioning of injection ports and vents.

The governing equation for the injection phase of these processes is derived from the conservation of mass of both the fluid and solid phase:

$$\nabla \cdot \left(h \frac{\mathbf{K}}{\mu} \nabla p \right) = \frac{\partial h}{\partial t} \tag{1}$$

where h is the thickness of the component, p is the fluid pressure, $\mathbf{K}$ is the permeability tensor and μ is the fluid viscosity. If thickness gradients are small enough to be neglected, then Eqn. 1 reduces to

$$\nabla \cdot \left(\frac{\mathbf{K}}{\mu} \nabla p \right) = \frac{\dot{h}}{h} \tag{2}$$

where $\dot{h} = dh/dt$. In RTM applications, $\dot{h} = 0$. In I/CM applications with rigid moulds, $\dot{h}$ will be constant throughout the part – it will be a known of the problem in velocity controlled compression, an unknown in the case of a force/pressure driven compression. In flexible mould / vacuum-bag processes, $\dot{h}$ will in general vary and be an unknown of the problem.

Inherent in Eqn. 2 is the conservation of (fluid) mass relation

$$\nabla \cdot \mathbf{q} = -\frac{\dot{h}}{h}, \tag{3}$$

where $\mathbf{q}$ is the Darcy velocity, with $\mathbf{q} = \phi\mathbf{v}$, and ϕ is the porosity, $\mathbf{v}$ being the fluid velocity, and Darcy's law for fluid flow,

$$\mathbf{q} = -\frac{\mathbf{K}}{\mu} \nabla p \tag{4}$$

The standard procedure for the numerical solution of Eqn. 1 (or 2) is to use a Control Volume method, whereby one grid of elements is used to evaluate fluid pressures, for example using the Galerkin Finite Element Method. A second grid (of control volumes) is then used to advance the fluid over some time interval. This ensures that fluid fluxes (pressure gradients) are evaluated *within* elements, and possible discontinuous pressure gradients at element boundaries are avoided. A large number of simulations have been carried out using this method, for RTM, I/CM, and flexible-bag processes, e.g. [1,2].

One problem with the Control Volume method, when used to simulate processes for which the Darcy velocity field is not divergence-free, for example I/CM and VARTM, is that resin mass is often not conserved on an element level, and this has consequences for the accuracy of the method. For example, for a linearly (P1) interpolated pressure p_{FE}, the FEM solution for $\mathbf{q}$ within any given element, $\mathbf{q}_{FE}$, is, from Eqn. 4,

$$\mathbf{q}_{FE} = -\frac{K}{\mu} \nabla p_{FE}, \tag{5}$$

a constant. Thus $\nabla \cdot \mathbf{q}_{FE} = 0$, and, from Eqn. 3, mass is not conserved within the element. Note that, in an RTM simulation (with constant permeability/thickness), where $\dot{h} = 0$, mass is conserved and this is not an issue.

Another approach is to use the so-called mixed methods, which yield a more accurate velocity and a locally conservative fluid mass. Here, both Eqns. 3 and 4 are discretised and a solution for both *p* and **q** is sought simultaneously, on either a single grid or on overlapping grids. The commonest scheme is to take *p* constant and **q** to vary linearly over an element/volume. The velocity obtained is more accurate than that using the standard Galerkin FEM with the CV scheme, but the mixed methods are computationally much more expensive.

An attractive alternative to these approaches is to use a *single* grid of finite elements. When non-conforming elements are used, essential mass conservation properties are encompassed. These elements are discussed in the next section.

ELEMENTS WITH CONSERVED MASS

Nonconforming (and conforming) P1 elements have been used to simulates I/CM processes (e.g. [3]) and have been shown to perform well. The performance of conforming and nonconforming P1 elements in IC/M and VARTM processes can be improved using a device introduced by Chou and Tang [4]. Here, the flux **q** is first approximated over an element *E* by the linear function $\mathbf{q}_a$, using a Taylor series expansion about the barycentre $\mathbf{x}_B$ of the element,

$$\mathbf{q}_a(\mathbf{x}) = \mathbf{q}_a(\mathbf{x}_B) + \left.\frac{\partial \mathbf{q}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_B} (\mathbf{x} - \mathbf{x}_B), \qquad \mathbf{x} \in E\,. \tag{6}$$

Assuming that $\mathbf{q}_a$ varies over the element according to

$$\mathbf{q}_a(\mathbf{x}) = \begin{bmatrix} r + sx \\ t + sy \end{bmatrix}, \tag{7}$$

then

$$\left.\frac{\partial \mathbf{q}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_B} (\mathbf{x} - \mathbf{x}_B) = s(\mathbf{x} - \mathbf{x}_B) = \frac{1}{2}(\nabla \cdot \mathbf{q}_a)(\mathbf{x} - \mathbf{x}_B) \tag{8}$$

The conservation of mass requirement is then

$$\int_E \nabla \cdot \mathbf{q}_a dS = \int_E f\,dS = \Delta f_E \tag{9}$$

where Δ is the area of the element and

$$f = -\frac{\dot{h}}{h}, \tag{10}$$

so that f_E is the average of *f* over the element, or, equivalently, $\nabla \cdot \mathbf{q}_a = f_E$. Thus

$$\mathbf{q}_a(\mathbf{x}) = \mathbf{q}_a(\mathbf{x}_B) + \frac{f_E}{2}(\mathbf{x} - \mathbf{x}_B), \qquad \mathbf{x} \in E . \tag{11}$$

Assuming one has first computed the flux $\mathbf{q}_{FE}$ using the standard Galerkin FEM, as in (5), one has

$$\mathbf{q}_a(\mathbf{x}) = -\frac{K}{\mu}\nabla p_{FE} + \frac{f_E}{2}(\mathbf{x} - \mathbf{x}_B), \qquad \mathbf{x} \in E \tag{12}$$

The second term on the right here is the correction to the FEM solution which ensures conservation of mass. It depends only on the instantaneous value of $\dot{h}/h$ and the element geometry, and so is the same for both conforming and non-conforming elements of the same geometry.

Continuity of Flux across Nonconforming Element Boundaries

Although the formulation described above ensures that mass is conserved over an element, there is no guarantee that the flux obtained is continuous across element boundaries. For the case of nonconforming linear triangular elements, the continuity of flux across element boundaries can be guaranteed by writing

$$\mathbf{q}_a(\mathbf{x}) = -\frac{K}{\mu}\nabla p_{FE} + \frac{f_E}{2}(\mathbf{x} - \mathbf{x}_B) + \mathbf{C}_E, \qquad \mathbf{x} \in E \tag{13}$$

where $\mathbf{C}_E$ is a small constant correction term [4]. It can be shown that

$$\mathbf{C}_E = \frac{1}{2}\begin{bmatrix} x_B f_E - \frac{1}{\Delta}\sum_{i=1}^{3} \bar{x}_i \int_E f N_i dS \\ y_B f_E - \frac{1}{\Delta}\sum_{i=1}^{3} \bar{y}_i \int_E f N_i dS \end{bmatrix} \tag{14}$$

where (x_B, y_B) are the barycentre coordinates, Δ is the area of the element, $(\bar{x}_i, \bar{y}_i)$ are the coordinates of the *vertices* of the element, and N_i are the three non-conforming element shape functions. Since $\int_E N_i dS = \Delta/3$, this implies that so long as f is *constant* along the mould, which is often the case in practice, $\mathbf{C}_E$ is zero and the flux is continuous, otherwise the correction term needs to be included. It was shown in some recent work [5] that a *regular* grid of right-sided elements, $\mathbf{C}_E$ is zero also for the case of *linearly* varying $\dot{h}$.

NONCONFORMING QUADRILATERAL ELEMENTS

Much research has been carried out recently into finite element analysis with non-conforming elements, in particular with quadrilateral elements, e.g. [6] (see [7]). This allows for a more powerful general meshing of moulds, using arbitrary arrangements of triangular and/or quadrilateral non-conforming elements.

As an illustrative example, consider the following simple problem: a square-shaped mould of length l and width w contains a uniform fibrous material initially filled with resin to $w \times l_1$. The upper mould is brought down at a constant velocity. The following data is used (the subscript "1" denotes values at the start of the simulation):

$$\dot{h} = -10^{-4}\,\text{m/s}\,(6\text{mm/min})$$
$$h_1 = 4\text{mm},\ \phi_1 = 0.5,\ \mu = 0.1$$
$$l_1 = 0.2\text{m},\ l = 0.4\text{m},\ w = 0.2\text{m}$$

The Carman-Kozeny relation was used to relate permeability to volume fraction (thickness):

$$K = \frac{d^2}{16k}\frac{(1-V_f)^3}{V_f^2}, \tag{15}$$

with $d = 10\times10^{-6}$, $k = 3.125\times10^{-3}$.

The problem was solved in six different ways, using piecewise linear triangular elements and quadrilateral elements:

- T1: triangle, single grid, conforming elements (with mass conservation)
- T2: triangle, single grid, non-conforming elements (with mass conservation)
- T3: triangle, control volumes
- Q1: quadrilateral, single grid, conforming elements
- Q2: quadrilateral, single grid, non-conforming elements
- Q3: quadrilateral, control volumes

The interpolation functions used for the case Q2 are given in the Appendix. Results for the fill-time were compared with the exact solution

$$T = -\frac{h_1\phi_1}{\dot{h}}\frac{l-l_1}{l} \tag{16}$$

and are shown in Fig. 1. The plot of percentage error against number of degrees of freedom (nodes) shows that the nonconforming triangular element with the mass conservation correction term performs well. Also, the non-conforming quadrilateral element performs satisfactorily when compared with the control volume method.

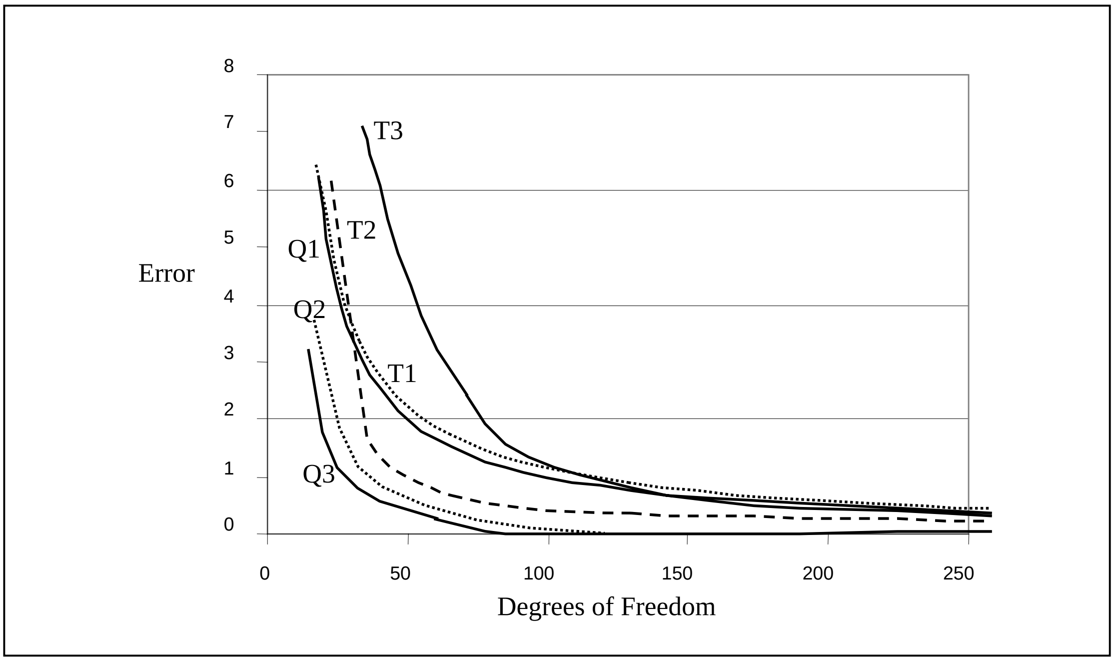

Fig. 1 % Error of IC/M flat-plate filling using different elements

CONCLUSIONS

In this study, the governing equations of LCM were solved using a number of different variants of the Finite Element Method. In particular, solutions were obtained using single-grid schemes with and without mass conservation, and with the standard CV method, involving triangular and quadrilateral elements. Numerical experiments were conducted to gauge the accuracy of the various schemes against standard solutions. It was demonstrated that triangular non-conforming elements with a mass-conservation correction term perform well in I/CM simulations, as do nonconforming quadrilateral elements, showing that simple single-grid meshes can be used productively for LCM simulations.

APPENDIX

The standard bilinear (conforming) Q4 element has as span $\{1, x, y, xy\}$. This span cannot be used for a nonconforming element since it cannot generate interpolation functions which are zero or 1 at the element mid-sides. Rotating a rectangle by 45 degrees, or equivalently, using the span $\{1, x, y, x^2 - y^2\}$ allows one to generate the interpolation functions. This can be amended to $\{1, x, y, x^2 - \frac{5}{3}x^4, y^2 - \frac{5}{3}y^4\}$ for greater accuracy [8]. This leads to the shape functions, in terms of natural coordinates (ξ, η), such that the four nodes of each element are located at $(\xi, \eta) = (\pm 1, \pm 1)$,

$$N_1(\xi,\eta) = \frac{1}{4} + \frac{1}{2}\xi - \frac{3}{8}\left[\left(\xi^2 - \frac{5}{3}\xi^4\right) - \left(\eta^2 - \frac{5}{3}\eta^4\right)\right]$$
$$N_2(\xi,\eta) = \frac{1}{4} + \frac{1}{2}\eta + \frac{3}{8}\left[\left(\xi^2 - \frac{5}{3}\xi^4\right) - \left(\eta^2 - \frac{5}{3}\eta^4\right)\right]$$
$$N_3(\xi,\eta) = \frac{1}{4} - \frac{1}{2}\xi - \frac{3}{8}\left[\left(\xi^2 - \frac{5}{3}\xi^4\right) - \left(\eta^2 - \frac{5}{3}\eta^4\right)\right]$$
$$N_4(\xi,\eta) = \frac{1}{4} - \frac{1}{2}\eta + \frac{3}{8}\left[\left(\xi^2 - \frac{5}{3}\xi^4\right) - \left(\eta^2 - \frac{5}{3}\eta^4\right)\right]$$

REFERENCES

1 A. Saouab, J. Breard, P. Lory, B. Gardarein and G. Bouquet “Injection simulations of thick composite parts manufactured by the RTM process”, *PComp. Sci. Tech.*, Vol.61, 2001, pp.445-451.

2 X. Song, “Vacuum Assisted Resin Transfer Molding (VARTM)”Model development and verification”, *PhD Thesis*, Virginia Polytechnic Institute and State University, 2003.

3 X-T. Pham and F. Trochu, “Simulation of Compression Resin Transfer Molding to Manufacture Thin Composite Shells”, *Polymer Comp.*, Vol.20, 1999, pp.436-459.

4 S-H. Chou and S. Tang, “Conservative P1 conforming and nonconforming Galerkin FEMS: effective flux evaluation via a nonmixed approach, SIAM J. Numer. Anal, Vol.38, 2000, pp. 660-680.

5 P.A. Kelly and S. Bickerton “Numerical simulation of liquid composite molding processes”, *Proc. Of the fifth Canadian International Composites Conference (CANCOM)*, 2005, pp.16-19.

6 C. Park and D. Sheen, “P1-nonconforming quadrilateral finite element methods for second-order elliptic problems’, *SIAM J. Numer Anal*, Vol.41, 2003, pp. 624-640.

7 Z. Chen, “Finite Element Methods and their applications’, pub. *Springer*, 2005.

8 Z. Cai, J. Douglas and X. Ye, “A stable nonconforming quadrilateral finite element method for the stationary stokes and Navier-Stokes equations, *Calcolo*, Vol.36, 1999, pp.215-232.