Fibre Buckling and Wrinkling in Plane Longitudinal Shear Flows of Ideal Fibre-Reinforced Materials

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Introduction

Thermoplastic composites are often processed by heating and shaping prepregs.Hence to understand the forming process for these materials a reliable model for the molten composite has to be formulated.It has been suggested [1] that such a model can be obtained by extending the continuum theory for fibre-reinforced solids initiated by Adkins and Rivlin [2] to fluids.This approach is based on an incompressible Newtonian liquid reinforced by inextensible fibres and is termed the Ideal Fibre-Reinforced Fluid model.

One of the problems encountered during the formation of composite components is the deviation of the fibres from the axial direction.The deviations can be divided into two types:those which occur at right angles to the shear planes (buckles) and those which occur in the shear planes (wrinkles).Such perturbations can be studied using a stability analysis.The continuum model outlined above is particularly well suited for such an analysis because it allows kinematic and rate dependent phenomena to be examined and fibre instabilities are known experimentally to depend on the rate of deformation. [3,4]

Boundary conditions can also play a crucial role in determining whether and what instabilities can occur.In this paper two different conditions will be examined.First the effect of a traction free surface on buckling instability is analysed.The second case deals with the effect of resin rich layers.These have been found experimentally [5] and used in some analytic predictions. [6,7] Such layers reduce shear stress by lubricating the flow and thus can allow wrinkles to develop under certain circumstances.

§1 Governing Equations

In the simple continuum model proposed the liquid is assumed to be incompressible.This constraint gives rise to the equation

$$\partial u/\partial x \;+\; \partial v/\partial y \;+\; \partial w/\partial z \;=\; 0 \qquad (1)$$

Here x,y and z represent a cartesian coordinate system in which u,v and w are the components of velocity.By using index notation this equation may be written in the more compact form

$$\partial u_i/\partial x_i \;=\; \partial u_1/\partial x_1 \;+\; \partial u_2/\partial x_2 \;+\; \partial u_3/\partial x_3 \;=\; 0 \qquad (2)$$

The index i takes the values 1,2 and 3 and as is conventional a repeated index denotes summation over all possible values of the index.In this notation x_1,x_2 and x_3 are the cartesian coordinates (equivalent to x,y and z) and u_1,u_2 and u_3 are the associated velocity components (equivalent to u,v and w).

The fibres shall be assumed to convect with the liquid during the flow i.e. an element of liquid will remain alongside the same section of fibre throughout the flow.This idea has been used in the context of solid composites and can be expressed [8] by the equation

$$\partial a_i/\partial t \;+\; u_j\partial a_i/\partial x_j \;=\; a_j\partial u_i/\partial x_j \qquad (3)$$

where t represents time and a_i are the components of a unit vector denoting the direction of the fibres.

The second kinematic constraint of fibre inextensibility ($a_ia_i = 1$) yields the equation

$$a_ia_jD_{ij} \;=\; 0 \qquad (4)$$

where the components of the rate of deformation tensor D_{ij} are defined as

$$D_{ij} \;=\; \tfrac{1}{2}(\partial u_i/\partial x_j \;+\; \partial u_j/\partial x_i) \qquad (5)$$

To complete the description of the molten composite a constitutive equation relating stress to the rate of strain is required. There is some evidence [9] to suggest that molten thermoplastic composites exhibit a Newtonian response during shearing flows, after an initial yield stress has been overcome. Thus a good model for the molten composite is that of a linear viscous fluid reinforced by inextensible fibres. The constitutive equation for such an anisotropic fluid is [1]

$$T_{ij} = -\Pi\,\delta_{ij} + T a_i a_j + 2\eta_T D_{ij} + 2(\eta_L - \eta_T)(a_i a_k D_{kj} + a_j a_k D_{ki}) \qquad (6)$$

T_{ij} are the components of the stress tensor and δ_{ij} is the Kronecker delta which is defined as :

$$\delta_{11} = \delta_{22} = \delta_{33} = 1 \qquad \delta_{ij} = 0 \quad \text{when} \quad i \neq j$$

Two distinct viscosities arise from the highly anisotropic nature of the fluid. The longitudinal viscosity (η_L) and transverse viscosity (η_T) are associated with shear along and perpendicular to the fibre direction respectively. It should be noted that although the molten composite is being modelled as a reinforced Newtonian fluid the constitutive equation (6) is highly non-Newtonian because of the anisotropy introduced by the fibres.

The pressure, Π, and the fibre tension, T, are arbitrary functions representing the reactions to the kinematic constraints of incompressibilty and inextensibility respectively. They are arbitrary in the sense that their values cannot be obtained from the constitutive equation. Instead they are determined by solving the equations of motion with appropriate boundary conditions. If the effects of body forces (e.g. gravity) are ignored then the three equations of motion take the form

$$\partial T_{ij}/\partial x_j = \rho(\partial u_i/\partial t + u_j\,\partial u_i/\partial x_j) \qquad (7)$$

where ρ is the density.

§2 Shear Flows

Molten composites can flow by shearing along or transverse to the fibre direction.This paper is concerned with longitudinal shear flows in unidirectionally reinforced composites in which the fibre direction and velocity are of the form

$$\begin{aligned} \boldsymbol{a} &= (0,0,1) \\ \boldsymbol{u} &= (0,0,w(x)) \end{aligned} \tag{8}$$

The fibres lie along the z-axis and the x-axis is perpendicular to the fluid surface.The axial stress gradient $\partial T_{zz}/\partial z$ is assumed to be zero so that the velocity assumes a linear profile.This is called a longitudinal plane Couette flow.

Units of length are chosen so that the flow occurs between two parallel plates at $x = 0$ and $x = -1$ which are moving with velocities $w = 0$ and $w = -V$ respectively.The usual no-slip conditions are assumed to hold at the plates.The full solution for such a flow is

$$\begin{aligned} \boldsymbol{u} &= (0,0,Vx) \\ \boldsymbol{a} &= (0,0,1) \\ T_{xx} &= T_{yy} = -\Pi(z) \\ T_{zz} &= -\Pi + T = m(x,y) \\ T_{xy} &= T_{yz} = 0 \\ T_{xz} &= \eta_L V \end{aligned} \tag{9}$$

The pressure Π is at most a function of z which is fixed by specifing either of the normal stress components T_{xx} or T_{yy}.The presence of the arbitrary function $m(x,y)$ allows for the possibility of the fibre tension varying in the x and y directions.The arbitrariness in the tension is removed when the remaining normal stress component T_{zz} is specified at one end (z = constant) of the flow region.

Shearing flows of this type can also occur if the upper plate is removed to give a traction free surface at $x = 0$.This boundary condition imposes a discontinuity in the shear stress at $x = 0$ which can only be equilibrated by having singular fibres at the surface.Singular fibre surfaces carry a finite force F but an infinite stress and are a feature associated with inextensible fibres.They have been investigated in the context of fibre-reinforced solids by several authors. [10,11] The singular fibre surfaces of the idealised model manifest themselves in real composites as regions of high stress concentration across which the stress changes rapidly.

To satisfy the zero traction boundary condition the stress components of solution (9) have to be modified to the following form

$$T_{xx} = T_{yy} = 0$$

$$T_{zz} = T = \eta_L Vz\delta(x) + m(x,y)$$

$$T_{xy} = T_{yz} = 0 \qquad (10)$$

$$T_{xz} = \eta_L V[1 - H(x)]$$

$$F = \eta_L Vz$$

where $\delta(x)$ is the Dirac delta function and $H(x)$ is the unit step function.

It should be noted that since the force in the singular fibre layer is linear in z the surface fibres change from being in compression to being in tension at $z = 0$.

§3 Perturbed Flow and Stability

The flow situations described by equations (9) and (10) are based on the assumption that the fibres are all aligned exactly parallel to the z-axis.However it has been noted [12] that in practice appreciable misalignments are present in nominally unidirectional samples.For the forming of composite components from molten prepregs it is important to know whether these deviations will significantly effect the end product.The orientation of fibres is particularly important since this often has a critical role in determining the compressive strength of the composite component. [13,14]

Small deviations from the ideal flow can be analysed using perturbation theory.Using this theory the evolution of perturbations with time can be studied.If they can be shown to die away with time then the unperturbed solution is stable.However,if the perturbations grow with time then considerable deviation from the basic flow and fibre direction can occur and the basic flow is unstable.

The analysis assumes that initially ($t = 0$) there are small imperfections present in the system.The ensuing time dependent solutions are then expressed as series in a small non-dimensional parameter ϵ,in which the first term in each series represents the steady (time independent) basic flow.Hence the full solutions are written in the form

$$
\begin{aligned}
\boldsymbol{u}_{tot} &= \boldsymbol{u} + \epsilon \boldsymbol{u}'(x,y,z,t) + O(\epsilon^2) \\
\boldsymbol{a}_{tot} &= \boldsymbol{a} + \epsilon \boldsymbol{a}'(x,y,z,t) + O(\epsilon^2) \\
\Pi_{tot} &= \Pi + \epsilon \Pi'(x,y,z,t) + O(\epsilon^2) \\
T_{tot} &= T + \epsilon T'(x,y,z,t) + O(\epsilon^2) \\
F_{tot} &= F + \epsilon F'(x,y,z,t) + O(\epsilon^2)
\end{aligned}
\qquad (11)
$$

It should be noted that although the perturbed velocities are small compared with V the associated displacements may become large as time progresses.

For small deviations from the basic flow it can easily be shown that the first order fibre perturbation is of the form

$$\boldsymbol{a}' = (a',b',0)$$

The component a' represents buckles which concern deformations that are perpendicular to the shear planes.The component b' concerns fibre deformations in the shear planes.This phenomenum is termed wrinkling.

The first order perturbations shall be assumed to take the form

$$\boldsymbol{u}' = \text{Real}\ \{\ \boldsymbol{u}^*(x,z)\ \exp(pt)\ \}$$

$$\boldsymbol{a}' = \text{Real}\ \{\ \boldsymbol{a}^*(x,z)\ \exp(pt)\ \}$$

$$\Pi' = \text{Real}\ \{\ \Pi^*(x,z)\ \exp(pt)\ \} \qquad (12)$$

$$T' = \text{Real}\ \{\ T^*(x,z)\ \exp(pt)\ \}$$

$$F' = \text{Real}\ \{\ F^*(x,z)\ \exp(pt)\ \}$$

The real part of the complex number p gives information about the stability of the flow.If it is greater than zero then the perturbations will grow exponentially large as time increases and the basic solution is unstable.If the real part of p is less than zero then the basic solution is said to be stable to this perturbation mode because the distortions die off with time.

The linearised governing kinematic equations are obtained by substituting the solutions defined by equations (11) and (12) into equations (1),(3) and (4) and picking out those terms of order ϵ.This gives

$$\partial u^*/\partial x + \partial w^*/\partial z = 0 \qquad (13)$$

$$pa^* + Vx\ \partial a^*/\partial z = \partial u^*/\partial z \qquad (14)$$

$$pb^* + VX\ \partial b^*/\partial z = \partial v^*/\partial z \qquad (15)$$

$$\partial w^*/\partial z + Va^* = 0 \qquad (16)$$

In a similar manner the stress components corresponding to these perturbations are obtained from the constitutive equation (6).

$$T^*_{xx} = -\Pi^* + 2\eta_T\, \partial u^*/\partial x + 2V(\eta_L - \eta_T)a^*$$

$$= -\Pi^* + 2\eta_L\, \partial u^*/\partial x$$

$$T^*_{yy} = -\Pi^*$$

$$T^*_{zz} = -\Pi^* + T^* + 2(2\eta_L - \eta_T)\partial w^*/\partial z + 2V(\eta_L - \eta_T)a^*$$

$$= -\Pi^* + T^* + 2\eta_L\, \partial w^*/\partial z \qquad (17)$$

$$T^*_{xy} = \eta_T\, \partial v^*/\partial x + V(\eta_L - \eta_T)b^*$$

$$T^*_{xz} = Ta^* + \eta_L(\partial w^*/\partial x + \partial u^*/\partial z)$$

$$T^*_{yz} = Tb^* + \eta_L\, \partial v^*/\partial z$$

It is assumed that the boundary conditions for the unperturbed flow are such that the unperturbed fibre tension T is a constant. The equations of motion then yield

$$-\partial\Pi^*/\partial x + 2\eta_L\, \partial^2 u^*/\partial x^2 + \eta_L(\partial^2 u^*/\partial z^2 + \partial^2 w^*/\partial x\partial z) +$$
$$T\, \partial a^*/\partial z = \rho(pu^* + Vx\, \partial u^*/\partial z) \qquad (18)$$

$$\eta_T\, \partial^2 v^*/\partial x^2 + \eta_L \partial^2 v^*/\partial z^2 + V(\eta_L - \eta_T)\partial b^*/\partial x +$$
$$T\, \partial b^*/\partial z = \rho(pv^* + Vx\, \partial v^*/\partial z) \qquad (19)$$

$$-\partial\Pi^*/\partial z + \partial T^*/\partial z + \eta_L(\partial^2 w^*/\partial x^2 + \partial^2 w^*/\partial z^2) +$$
$$T\, \partial a^*/\partial x = \rho(pw^* + Vu^* + Vx\, \partial w^*/\partial z) \qquad (20)$$

§4 Fibre Buckling

It is apparent from the foregoing equations that the two types of fibre perturbation do not interact and hence they can be discussed separately.In this section fibre buckling will be analysed with no fibre wrinkling so that

$$b^* = 0 \tag{21}$$

It is also reasonable to suppose that there is no component of the perturbed velocity in the y direction and thus

$$v^* = 0 \tag{22}$$

This choice of b^* and v^* ensures that the kinematic equation (15) and the second equation of motion (19) are satisfied.The three remaining kinematic equations (13),(14) and (16) are relations between a^*,u^* and w^*.It is possible to obtain solutions for these quantities in terms of an unknown function of x,f(x),and an unknown function of z,g(z).The solutions are

$$a^* = -dg/dz \tag{23}$$

$$u^* = -pg(z) - Vx\ dg/dz \tag{24}$$

$$w^* = V(\ g(z) + f(x)/p\ ) \tag{25}$$

These equations can be substituted into the first equation of motion to give

$$\partial\Pi^*/\partial x + (T + p\eta_L)\ d^2g/dz^2 + V\eta_L x\ d^3g/dz^3 = \rho(\ p^2g(z) + 2pVx\ dg/dz + V^2x^2\ d^2g/dz^2\ ) \tag{26}$$

If it is assumed that the perturbed pressure gradient $\partial\Pi^*/\partial x$ depends on x then the terms independent of x can be equated to give

$$(T + p\eta_L)\ d^2g/dz^2 = \rho p^2g(z) \tag{27}$$

This has periodic solutions of the form

$$g(z) = C_1 \cos (Kz + \alpha) \tag{28}$$

where α and C_i,i = 1,2,3,.. are constants here and in all subsequent expressions.These solutions represent buckles of wavelength $(2\pi/K)$ if K^2 is positive,where

$$K^2 = \frac{-\rho p^2}{T + p\eta_L} \tag{29}$$

This may be regarded as a quadratic equation in p which has the solutions

$$p = \frac{-\eta_L K^2 \pm (\eta_L{}^2 K^4 - 4K^2 T\rho)^{\frac{1}{2}}}{2\rho} \tag{30}$$

Therefore when the tension in the fibres T is positive both of these values of p have negative real parts.Then since the buckles depend exponentially on time as given in equation (12) they will decay in magnitude as time increases and the system will be stable with respect to sinusoidal fibre buckles.Any sinusoidal buckles which are present initially will be damped out during the flow by the effect of the positive tension.It should also be noted that as the magnitude of the tension is increased they will decay at a faster rate.Hence it is desirable in practical situations to have as large a tension in the fibres as is possible.

If the fibres are in compression then one of the values of p in (30) will have a negative real part and this will cause any sinusoidal buckles to grow as the flow evolves.In the case when the tension is exactly zero the fibre buckles will neither grow nor decay:any buckles which are initially present will remain unchanged during the remainder of the flow.

The perturbed stress component T^*_{yy} $(= -\Pi^*)$ can be obtained by equating all the terms which depend on x in equation (26).Upon integration with respect to x,it is found that

$$\Pi^* = Vx^2(\tfrac{1}{2}K^2\eta_L + p\rho)\, dg/dz - (\rho V^2K^2x^3/3)\, g(z) + l(z) \tag{31}$$

The function of integration l(z) can be determined by applying the appropriate boundary conditions at the traction free surface.If it is assumed that the small perturbations do not give rise to any tractions above the bounding fibre,then it is shown in App. 1 that the appropriate conditions are

$$-\Pi^* = V\eta_L z\, \partial a^*/\partial z \qquad \text{at} \quad x = 0 \tag{32}$$

$$\eta_L(\partial u^*/\partial z + \partial w^*/\partial x) = dF^*/dz \qquad \text{at} \quad x = 0 \tag{33}$$

The second of these equations can be used to find the singular force which results from the perturbation.

The perturbed axial velocity w^* is given in terms of the unknown function f(x).This may be determined by equating the terms independent of z in the third equation of motion (20).

$$f(x) = C_2 \exp\left[(\rho p/\eta_L)^{\frac{1}{2}}x\right] + C_3 \exp\left[-(\rho p/\eta_L)^{\frac{1}{2}}\right] \qquad (34)$$

The perturbed axial stress T^*_{zz} is obtained to within an arbitrary function $m^*(x,y)$ by integrating the remaining terms in equation (20) with respect to z.The arbitrary function is fixed by applying boundary conditions at one end (z = constant) of the flow region.

The full solution can be expressed in terms of the functions f(x) and g(z) as follows:

$$a^* = -\,dg/dz \qquad b^* = c^* = 0$$

$$u^* = -pg(z) - Vx\,dg/dz \qquad v^* = 0 \qquad w^* = V(\,g(z) + f(x)/p\,)$$

$$T^*_{xx} = -Vx^2(\tfrac{1}{2}\eta_L K^2 + p\rho)dg/dz - 2V\eta_L dg/dz + K^2[V\eta_L z + (V^2x^3/3)]g(z)$$

$$T^*_{yy} = -Vx^2(\tfrac{1}{2}\eta_L K^2 + p\rho)dg/dz + K^2[V\eta_L z + (V^2x^3/3)]g(z)$$

$$T^*_{zz} = \eta_L V\,dg/dz + m^*(x,y) \qquad (35)$$

$$T^*_{xy} = T^*_{yz} = 0$$

$$T^*_{xz} = -T\,dg/dz + \eta_L[\,(V/p)\,df/fx - p\,dg/dz + K^2Vx\,g(z)\,]$$

$$F^* = -p\eta_L g(z) + V(C_2 - C_3)(\rho\eta_L/p)^{\frac{1}{2}} z + C_4$$

The sinusoidal buckles in the fibres have caused a perturbation in the force acting in the singular fibre surface.This additional force consists of both a sinusoidal and a linear variation with z.

§5 Fibre Wrinkles

In this section linear shear flow between two flat plates shall be examined.The basic solution for this flow is given by equation (9) and no singular fibres are present because of the frictional forces at the top plate.The presence of the parallel flat plates will inhibit any buckling of the fibres and hence

$$a^* = 0 \tag{36}$$

Also if the fluid is to remain in contact with the plates at all times there must be no velocity in the x direction i.e.

$$u^* = 0 \tag{37}$$

With these results equations (13) and (20) yield the perturbed velocity in the z direction.

$$w^* = C_5 \exp\left[(\rho p/\eta_L)^{\frac{1}{2}}x\right] + C_6 \exp\left[-(\rho p/\eta_L)^{\frac{1}{2}}x\right] \tag{38}$$

In §4 the buckles were assumed to be sinusoidal and similarly the wrinkles shall be chosen to have a sinusoidal variation of wavelength $(2\pi/K)$ in the z direction.Then b^* and v^* may be expressed in the form

$$b^*(x,z) = b^o(x) \exp(iKz)$$

$$v^*(x,z) = v^o(x) \exp(iKz)$$

Using these expressions equation (15) simplifies to

$$(p + iKVx)b^o = iKv^o \tag{39}$$

This can then be used with equation (19) to give

$$\eta_T \frac{d^2v^o}{dx^2} + \frac{iKV(\eta_L - \eta_T)}{(p + iKVx)} \frac{dv^o}{dx}$$
$$\left[\frac{K^2V}{(p + iKVx)^2} - \frac{K^2T}{(p + iKVx)} - K^2\eta_L - \rho(p + iKVx)\right]v^o = 0 \tag{40}$$

Providied that p is non-zero this equation has a series solution of the form

$$v^o = M(1 + m_2x^2 + m_3x^3 + m_4x^4 + \ldots\ldots) + N(x + n_2x^2 + n_3x^3 + n_4x^4 + \ldots\ldots) \quad (41)$$

(The coefficients m_2, m_3 and n_2, n_3 are listed in App. 2.)

One possible explanation for the localised fibre wrinkling which has been reported in real samples [15] is the presence of a resin rich layer.Due to the lubricating effect of the resin a friction free layer can form in which no shearing stress can be supported.If this layer occurs at the top plate and is of depth γ the boundary conditions are

$$T_{xy} = T_{xz} = T_{yz} = 0 \quad \text{for} \quad -\gamma < x < 0$$

At $x = 0$ these conditions become

$$\frac{dv^o}{dx} + \frac{iVK(\eta_L - \eta_T)}{p\eta_T} v^o = 0 \quad (42)$$

$$dw^o/dx = 0 \quad (43)$$

$$v^o(p\eta_L + T) = 0 \quad (44)$$

The first and last of these conditions along with the series solution (41) give

$$\frac{N}{M} = \frac{-iVK(\eta_L - \eta_T)}{p\eta_T}$$

$$M(p\eta_L + T) = 0$$

To ensure a non-trivial solution p must take the value

$$p = -T/\eta_L \quad (45)$$

Thus when the fibres are in tension any sinusoidal wrinkles in the resin rich layer will decay,but when the fibres are in tension they will grow exponentially with time.

Condition (43) constrains the perturbed axial velocity to be of the form

$$w^* = C_5 \{\exp[(\rho p/\eta_L)^{\frac{1}{2}}x] + \exp[-(\rho p/\eta_L)^{\frac{1}{2}}x]\} \quad (46)$$

§6 Conclusions

In this paper the Ideal Fibre-Reinforced Fluid Model has been used to analyse the development of fibre misalignments during shear flows of nominally unidirectional molten composites.The mathematical model is in agreement with observations of real samples which have found two distinct types of fibre deviation.Using a stability analysis the model has been used to predict what stress should be applied to alleviate such misalignments.

In the two flows which have been examined the important factor for determining stability has turned out to be the fibre tension T.This is the difference between the axial and normal component of stress ($T = T_{zz} - T_{xx}$) and the importance of such stress component differences is characteristic of non-Newtonian fluids.The requirement for T to be positive for stability is in accordance with intuition and has been derived in other solutions of this type.[16] Furthermore the stability criteria are independent of the velocity V of the lower plate.

These results suggest that during the manufacture of composite components from molten pre-pregs it is important to ensure that the fibres remain in tension.Compresive forces in the fibres will cause any misalignments which are present to grow and so lead to poor alignment and reduced strength in the finished product.

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Appendices

App. 1

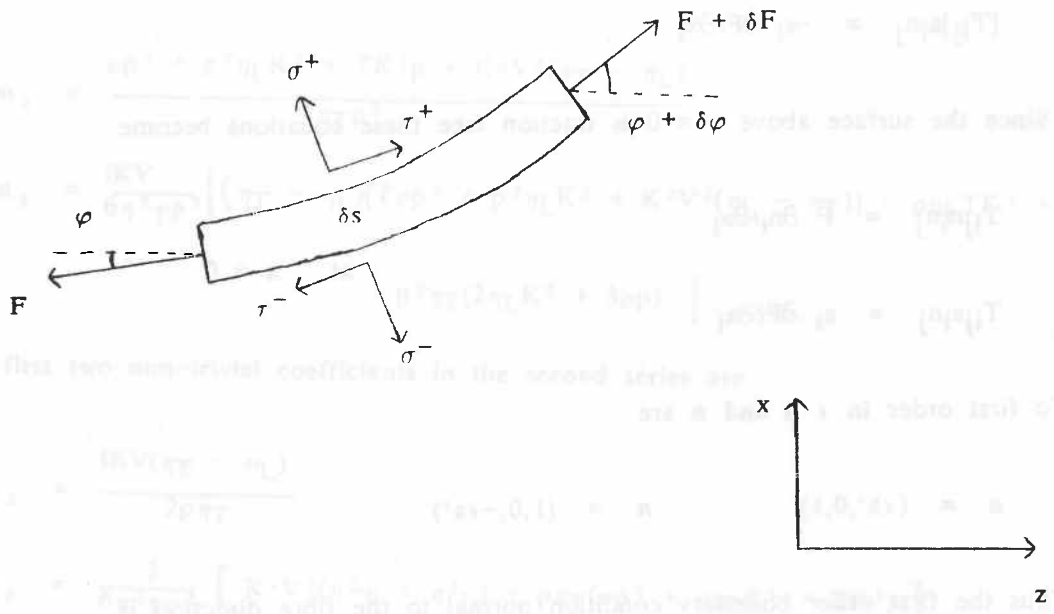

Fig. 1 Equilibrium of a Small Element

Consider a small section δs of the composite containing a fibre as shown in Fig. 1. The components of stress normal and tangential to the fibre direction are represented by σ and τ respectively. The superscripts $^+$ and $^-$ denote their values on the upper and lower sides of the element. The force in the fibre is F.

The element is assumed to be in equilibrium so that forces can be equilibrated along and normal to the fibre direction to give [17]

$$[\sigma] = F\, \partial n_i/\partial x_i$$

$$[\tau] = -a_i\, \partial F/\partial x_i$$

where the square brackets [] denote a jump in going from the upper to the lower side of the element and n_i are the components of the unit normal to the fibre direction. The components of stress can be expressed in their cartesian components by using the following expressions

$$[\sigma] = [T_{ij}]n_i n_j$$

$$[\tau] = [T_{ij}]a_i a_j$$

Hence the boundary conditions are

$$[T_{ij}]n_in_j \quad = \quad F\ \partial n_i/\partial x_i$$

$$[T_{ij}]a_in_j \quad = \quad -a_i\ \partial F/\partial x_i$$

Since the surface above x = 0 is traction free these equations become

$$T_{ij}n_in_j \quad = \quad F\ \partial n_i/\partial x_i$$

$$\text{at} \quad x = 0$$

$$T_{ij}a_in_j \quad = \quad a_i\ \partial F/\partial x_i$$

To first order in ϵ ***a*** and ***n*** are

$$\boldsymbol{a} \quad = \quad (\epsilon a',0,1) \qquad\qquad \boldsymbol{n} \quad = \quad (1,0,-\epsilon a')$$

Thus the first order boundary condition normal to the fibre direction is

$$-2T_{xz}a^* \quad + \quad T^*_{xx} \quad = \quad F\ \partial a^*/\partial z \qquad\qquad \text{at} \quad x = 0$$

By substituting the stress components and singular force this equation may be written as

$$-2\eta_L Va^* \quad + \quad 2\eta_L\ \partial u^*/\partial x \quad - \quad \Pi^* \quad = \quad V\eta_L z\ \partial a^*/\partial z \qquad \text{at} \quad x = 0$$

However

$$-2\eta_L Va^* \quad + \quad 2\eta_L\ \partial u^*/\partial x \quad = \quad 2\eta_L(\partial w^*/\partial z + \partial u^*/\partial x) \quad = \quad 0$$

and hence the condition becomes

$$-\Pi^* \quad = \quad V\eta_L z\ \ \partial a^*/\partial z \qquad\qquad \text{at} \quad x = 0$$

To first order in ϵ the second condition is

$$a^*(T^*_{xx} - T^*_{zz}) \quad + \quad T^*_{xz} \quad = \quad \eta_L(\partial u^*/\partial z + \partial w^*/\partial x) \quad = \quad dF^*/dz$$

$$\text{at} \quad x = 0$$

App. 2

The first two non-trivial coefficients in the first series are

$$m_2 = \frac{\rho p^3 + p^2\eta_L K^2 + TK^2 p + K^2V^2(\eta_T - \eta_L)}{2\eta_T p^2}$$

$$m_3 = \frac{iKV}{6\eta^2{}_T p^3}\Big[(\eta_T - \eta_L)[T\rho p^3 + p^2\eta_L K^2 + K^2V^2(\eta_L - \eta_T)] - p\eta_L TK^2 +$$

$$p^2\eta_T(2\eta_L K^2 + 3\rho p)\Big]$$

The first two non-trivial coefficients in the second series are

$$n_2 = \frac{iKV(\eta_T - \eta_L)}{2p\eta_T}$$

$$n_3 = \frac{1}{6\eta^2{}_T p^2}\Big[K^2V^2(\eta^2{}_T - \eta^2{}_L) + p\eta_T(\rho p^2 + p\eta_L K^2 + TK^2)\Big]$$