

STUDY OF THE MECHANICAL BEHAVIOUR OF DIAPHRAGM FILMS

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ABSTRACT

A biaxial inflation experiment is presented to determine the biaxial properties of commercially available diaphragm materials. The pressure and deflection of the bubble are measured during testing. Extension ratios are determined after inflation. Results are presented for Upilex 125R film. A FORTRAN code is adapted to predict the deformations and extension ratios of the experiments. The code uses large deformation theory for incompressible isotropic elastic materials. Both the Mooney and neo-Hookean strain energy functions have been programmed into the code. The results from the code are presented and compared to experimental results where the polar extension ratio λ_{2p} is less than 1.5. Excellent agreement is found and an equal biaxial stress state exists in the polar region of the bubble.

INTRODUCTION

The advent of advanced forming technologies for thermoplastic composite materials has led to an interest in the diaphragm forming process. This process is used to manufacture high quality complex shaped parts suitable for use in aircraft structures. Some of the limitations to date on the diaphragm forming process have been the mechanical properties of the diaphragms. The requirements of the diaphragms for forming are high strain to failure and high strength as well as toughness. These properties can be obtained using the commercially available UBE Polyimide film Upilex R supplied by ICI films. However very little data exists about the mechanical properties of these films at the temperature ranges experienced during diaphragm forming.

The diaphragm forming process has been described by Mallon et al [1] and Okine [2], during the forming process stacked prepreg material is placed between two diaphragms, the diaphragms are clamped at the edges and a vacuum is exerted between the diaphragms. The diaphragms maintain biaxial tension on the laminate during deformation, thus restricting laminate buckling and wrinkling [1], and thus enhancing the quality of the formed part.

The object of this paper is to investigate the biaxial properties of Upilex 125R and to compare the measured experimental values with the predicted numerical values based on different strain energy function forms. The biaxial test takes place at the typical temperature of the diaphragm forming process and the results therefore provide hitherto unavailable information in relation to the response of Upilex R to deformation pressures at high

temperatures. It is hoped that this work and continuing research will help researchers build a better diaphragm forming model for advanced thermoplastic composites.

BACKGROUND THEORY

The use of biaxial inflation to measure properties of films has been employed for many years, and much work has been published in this area. Adkins and Rivlin [3] developed equations to describe the inflation of a plane circular membrane. Hart-Smith and Crisp [4] also present a numerical routine for solving the problem. Kirkland, Duncan and Haward [5] present a diaphragm test for sheet plastics while Schmidt and Carley [6] present experiments and the formulation of the inflation of heat softened polymeric sheets.

The theory of the inflation of a circular membrane is well documented in the literature and thus only a brief summary of the relevant equations will be presented here. Figure 1 shows the cylindrical coordinate system used to locate points on the middle surface of the bubble.

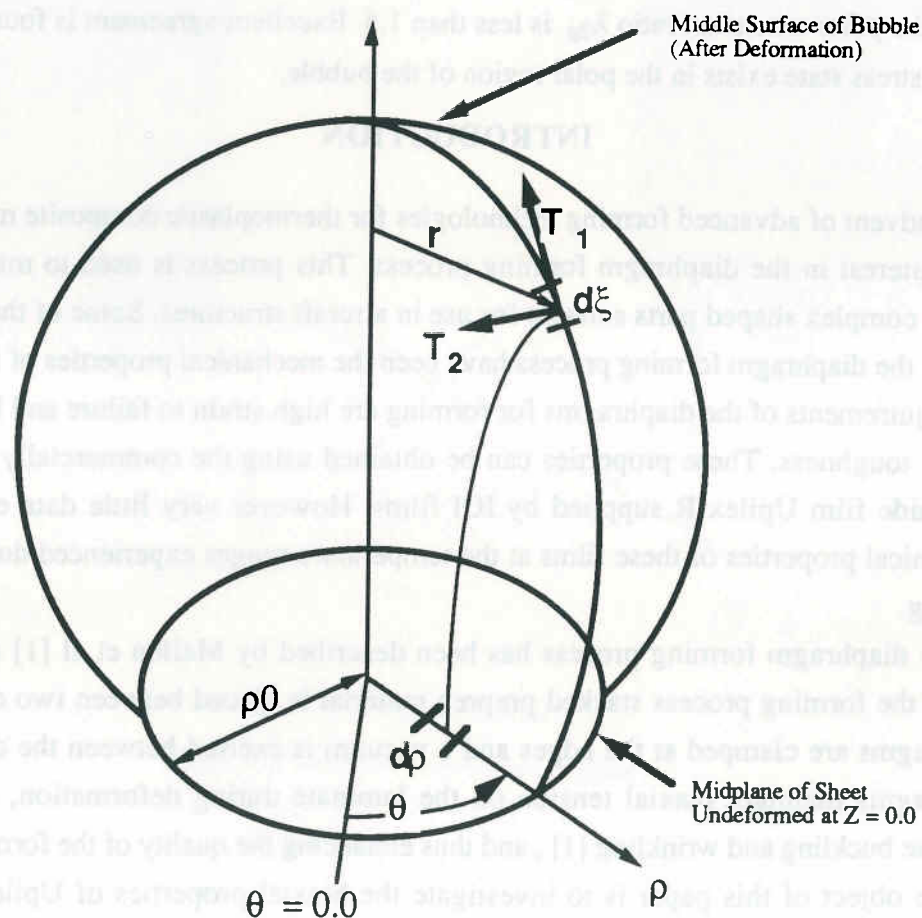


Figure 1. Cylindrical Coordinate Systems used in the analysis of the axially symmetric bubble deformation.

T_1 and T_2 are the meridional and circumferential stresses in the membrane and $d\xi$ is the deformed length of $d\rho$. The three sets of equations required to describe the problem are the equilibrium, geometric and constitutive equations. The static equilibrium equations assume a state of plane stress and the reduced equilibrium equations have the form :

$$\frac{d(T_1 \rho)}{d\rho} = T_2 \quad \text{and} \quad \kappa_1 T_1 + \kappa_2 T_2 = P \quad (1)$$

where P is the inflation pressure in the membrane and κ_1 and κ_2 are the meridional and circumferential curvatures respectively. The relationship $\partial T_2 / \partial \theta = 0$, which arises from symmetry, is also employed.

The geometrical relations of Codazzi presented by Love [7] are also employed to give the geometrical relationships

$$\frac{dr}{d\rho} = \pm \lambda_1 \sqrt{1 - (\kappa_2 r)^2} \quad \text{and} \quad \frac{dz}{d\rho} = \pm \lambda_2 \kappa_2 r \quad (2)$$

where the extensions ratios are defined as

$$\lambda_1 = \frac{d\xi}{d\rho}, \quad \lambda_2 = \frac{2\pi r}{2\pi \rho} \quad \text{and} \quad \lambda_3 = \frac{H}{H_0} \quad (3)$$

Where H and H_0 are the final and initial thickness values of the diaphragm film.

The stresses in the membrane T_1 and T_2 can be calculated in terms of λ_1 and λ_2 for a thin sheet of incompressible highly elastic material which is isotropic in its undeformed state. For such a material the strain energy function W is a function of I_1 and I_2 given in terms of the principle extension ratios λ_1 , λ_2 and λ_3 by the relations

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad \text{and} \quad I_2 = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} \quad (4)$$

$$\lambda_1 \lambda_2 \lambda_3 = 1 \quad (5)$$

The reduced form of the expressions for the principal components of the membrane stresses [3] are

$$T_1 = 2 \lambda_3 (\lambda_1^2 - \lambda_3^2) \left(\frac{\partial W}{\partial I_1} + \lambda_2^2 \frac{\partial W}{\partial I_2} \right) \quad (6)$$

$$T_2 = 2\lambda_3 (\lambda_2^2 - \lambda_3^2) \left(\frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2} \right) \quad (7)$$

while T_3 is assumed negligible. Different forms of the strain energy function have been proposed by various researchers the most significant of which is the Mooney form

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \quad (8)$$

Where C_{10} and C_{01} are constants. Another form of strain energy function, the neo-Hookean or Gaussian form given below, was also investigated

$$W = C_{10} (I_1 - 3) \quad (9)$$

Both these forms were incorporated into a FORTRAN coding as described in the next section.

NUMERICAL APPROACH

Using the theory presented in the section above a numerical routine can be developed to solve the equations and predict the bubble deformation and extension ratios for a given inflation pressure. These results are compared to a finite element model of the problem.

The author has adapted the FORTRAN code presented by Schmidt [8] to model the inflation of the circular membrane. The method of solution of the relevant equations is summarized below. The solution procedure starts at the pole of the bubble where the thickness ratio λ_3 is provided. And assuming that an equal biaxial state exists at the pole of the bubble, λ_2 and λ_1 are determined using equation 5. Hart-Smith and Crisp [4] have suggested a relationship between the curvature and extension at the pole

$$\kappa_p = \frac{2 \sqrt{(\lambda_p - 1)}}{\lambda_p} \quad (10)$$

where the subscript p denotes values at the pole, and this is used to approximate the initial value of curvature at the pole. The pole of the bubble is used as the starting point and the equations are solved for stress and curvature at the pole, the dimensionless radius is then incremented and the equations are solved using a second order Runge-Kutta technique described by Hart-Smith and Crisp [4]. All the derivatives at the pole are positive and the marching process proceeds to the equator of the bubble where the derivatives in r and κ_2 are

equal to zero. The equator is the most sensitive region in the entire calculation and Schmidt presents a method where by the step size in the routine is cut by a factor of 5 near the equator. This reduction in step size gives greater accuracy while avoiding numerical instabilities which could occur with smaller step sizes. The numerical code presented by Schmidt is very general and allows programming, using a subroutine, of different strain energy functions W . Like the routine presented by Hart-Smith and Crisp [4] the numerical code is simplified by having just one set of equations applied to the whole system. A formulation presented by Adkins and Rivlin [3] had separate equations for deformations near the pole and the equator.

The code was programmed and checked with the published results to ensure that no programming errors existed before applying the code to the specific problem and material at hand. The constants of the Mooney strain energy function were determined by obtaining a ratio of C_{10}/C_{01} to give a pole height similar to the experimental results. The program outputs the three extension ratios and the bubble profile along with the two non-zero principal stresses T_1 and T_2 . The code also calculates the constants associated with the different strain energy forms.

EXPERIMENTAL

The inflation test gives a true state of biaxial stress in the region of the bubble pole and thus was employed in this study. The tests on the Upilex material were carried out at the typical processing temperature of the APC-2 thermoplastic composite material and therefore a special high temperature test rig was used as shown in Figure 2 overleaf. The internal diameter of the clamp is 76.5mm. The pressurising medium, usually nitrogen gas is introduced through a hole in the plate on the underside of the film. The ring and bottom plate are clamped together using a simple nut and bolt arrangement. During the test the temperature of the film is measured using a thermocouple and the inflation pressure is measured using a calibrated pressure transducer which measures the line pressure between the bubble and the pressure supply. The central pole deflection of the bubble is measured using the arrangement shown in Figure 2. The LVDT is a special high temperature displacement transducer which has a moveable probe. The lever arrangement is used to reduce the force exerted on the bubble membrane by the probe of the LVDT. The lever is balanced so that there is a slightly positive force on the surface of the bubble.

The apparatus of Figure 2 is used in conjunction with the autoclave described by the authors [9]. The large heated area of the autoclave is needed to heat both the LVDT and the clamp to 380°C. Specimens of the Upilex material are cut from a roll and a grid is drawn on each specimen using a standard drawing pen. The grid arrangement is shown in Figure 3.

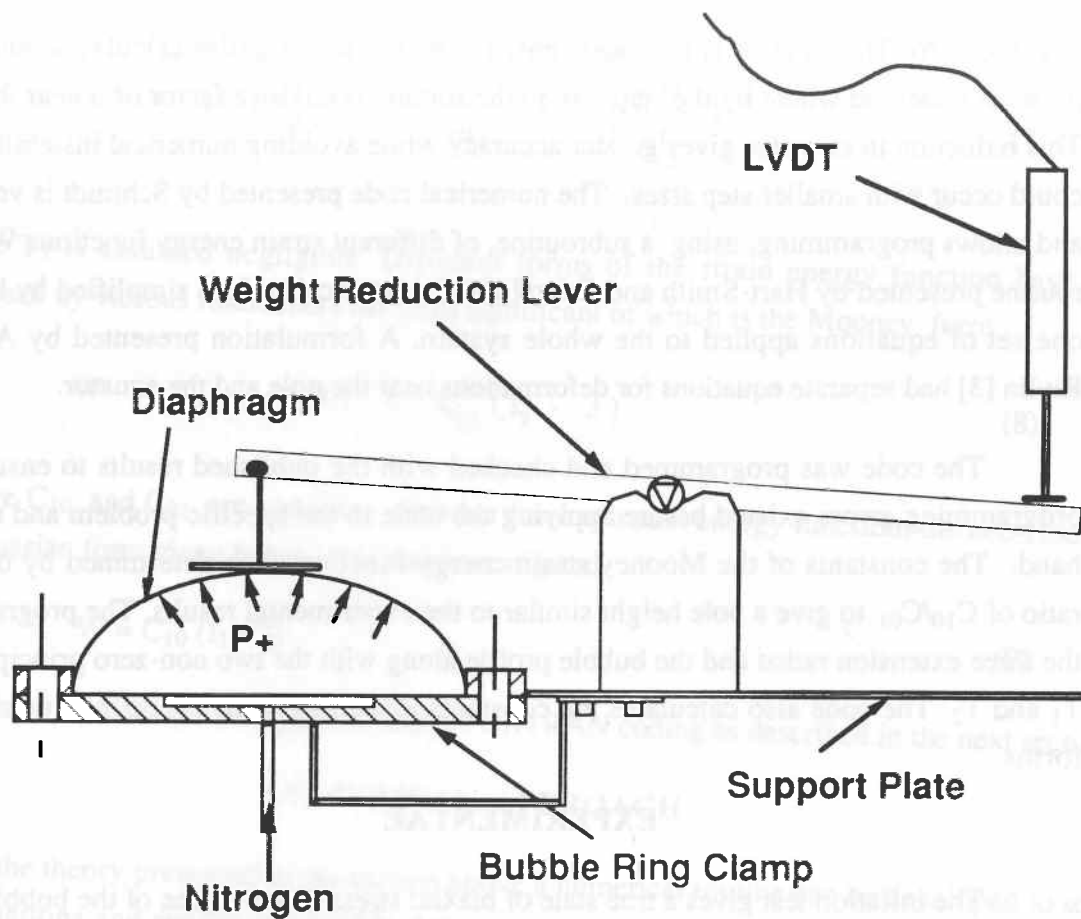


Figure 2. Biaxial Inflation Rig for Polymeric Films

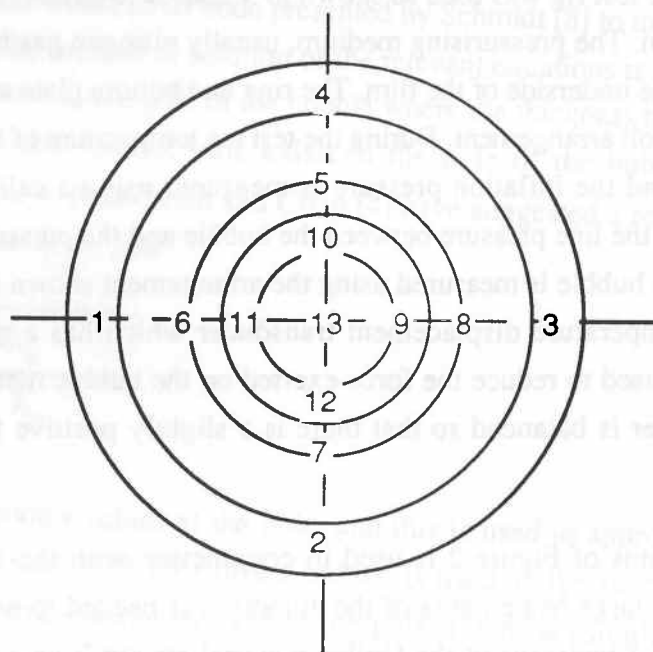


Figure 3. Typical grid pattern drawn on circular specimen before testing.
(Numbers indicate locations for thickness measurement)

The thickness of the specimen is measured at a number of different points as shown in Figure 3 and the diameters of the circle are also measured before inflation. The specimen is then placed in the clamp, tightened up and heated to the set point temperature. Once the bubble has reached this temperature the pressure is introduced gradually. The pressure and deflection of the bubble are measured simultaneously and recorded using a data acquisition system. Once the deflection has reached a desired value the pressure is held constant and the whole arrangement is cooled to room temperature. Final measurements are then taken of the thickness and the deformed circle diameters. A typical deflection versus pressure curve is shown in Figure 4 below

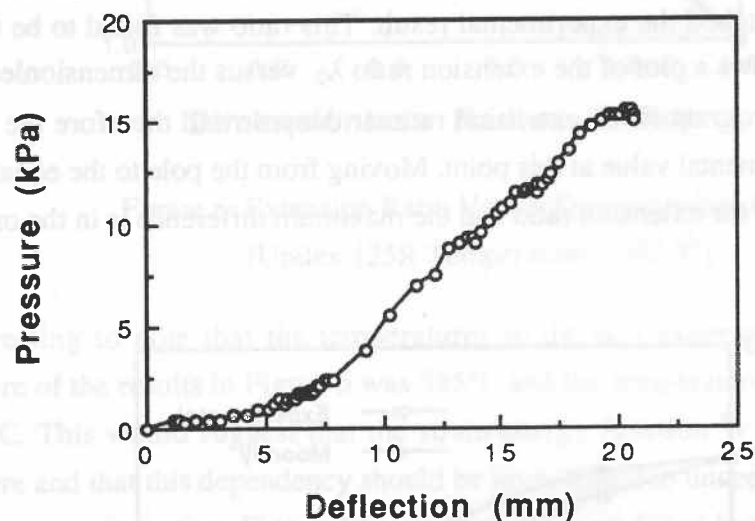


Figure 4. Pressure vs. Deflection Curve for Upilex 125R
(Temperature = 387°C, Max Defl = 20.639, Max Press = 15.51 kPa)

A summary of some experimental results is presented in Table 1 below. The thickness extension ratio at the pole, λ_{3p} , was measured and the assumption of equal biaxial extension at the pole was used to determine λ_{2p} .

Sample	Temp (°C)	Press. (kPa)	Defln (mm)	λ_{2p}	λ_{3p}
J171	388	17.42	18.041	1.14557	0.762
J175	389	22.686	24.203	1.2559	0.634
J178	386	24.837	27.156	1.3199	0.574
J181	387	15.51	20.639	1.2126	0.680
J183	388	18.14	17.425	1.1337	0.778
J185	385	17.90	21.845	1.2171	0.675

Table 1. Experimental Results

RESULTS

The results from the numerical routines are compared and presented with the experimental results. The first objective was to find a ratio of C_{10}/C_{01} for the Mooney strain energy function which would give a predicted bubble height equal to the experimental results for the same inflation pressures. A typical experimental result was chosen, the film temperature at inflation was 385°C and the inflation pressure used was 17.90 kPa. The recorded pole deflection of the bubble was 21.8 mm which is well below the hemispherical height of 38.25 mm. The FORTRAN code was then executed with varying ratios of C_{10}/C_{01} until a predicted pole deflection matched the experimental result. This ratio was found to be $C_{10}/C_{01} = 0.697$. Figure 5 below shows a plot of the extension ratio λ_2 versus the dimensionless radius. One of the inputs to the program is the extension ratio at the pole and therefore the numerical result matches the experimental value at this point. Moving from the pole to the equator the numerical results over-predict the extension ratio and the maximum difference is in the order of 2.2%.

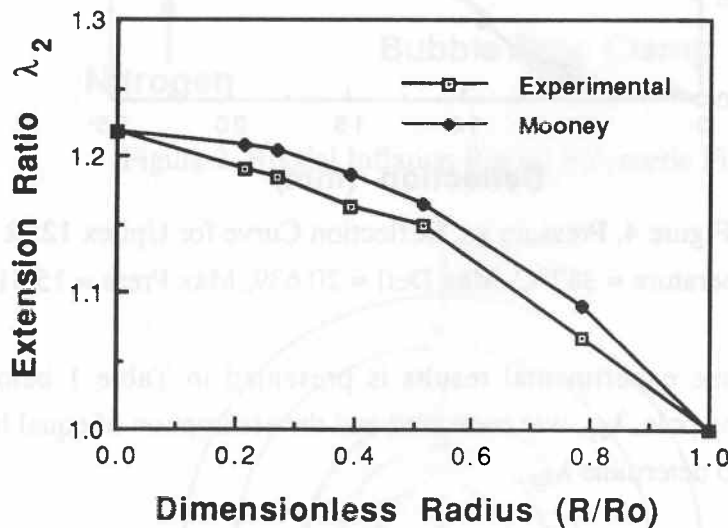


Figure 5. Extension Ratio versus Dimensionless Radius
(Upilex 125R Temperature = 385°C)

Another set of experimental results were now analysed using the same ratio of 0.697. Depending on the extension ratio at the pole and the inflation pressure the code will give different values of C_{10} and C_{01} . Figure 6 presents results for a bubble inflation which resulted in an extension ratio at the cap of 1.21 and a bubble height of 20.6mm with an inflation pressure of 15.51 kPa. As can be seen there is excellent agreement between the experimental and the numerically predicted results for λ_2 .

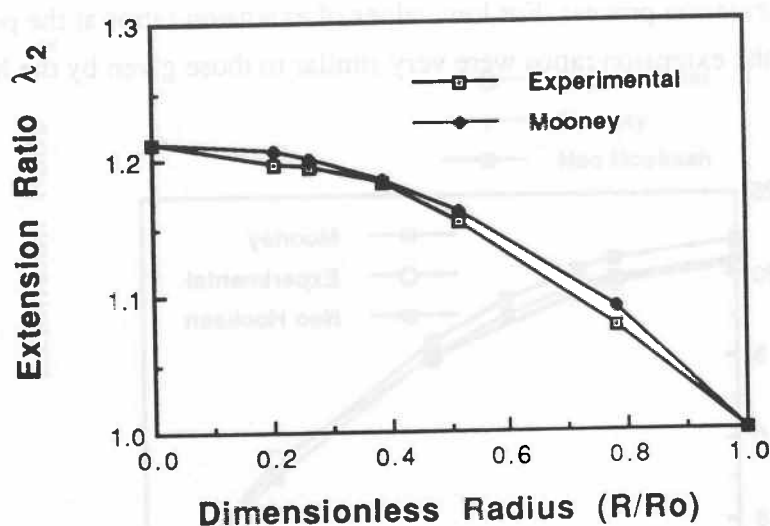


Figure 6. Extension Ratio Versus Dimensionless Radius
(Upilex 125R Temperature = 387°C)

It is interesting to note that the temperatures in the two experiments varied slightly, the temperature of the results in Figure 5 was 385°C and the temperature of the results in Figure 6 was 387°C. This would suggest that the strain energy function W is very dependant on the temperature and that this dependency should be investigated to understand the behaviour of the film as temperature varies. Figure 7 below shows the predicted bubble profile and the actual bubble profile for the experiment of Figure 6.

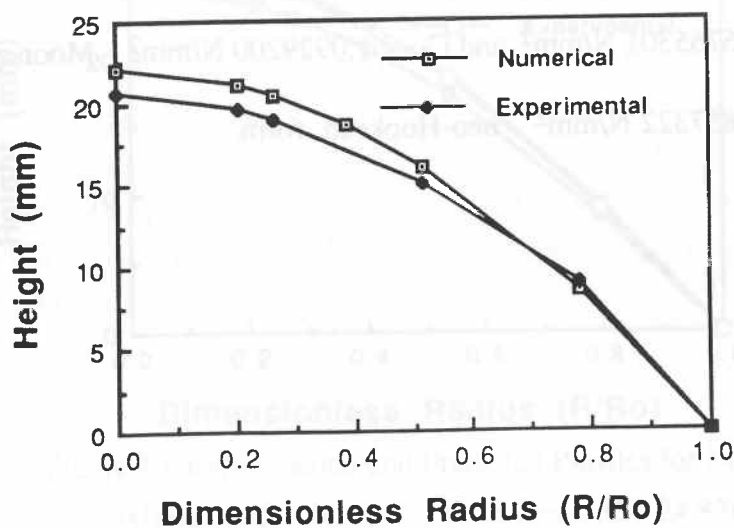


Figure 7. Predicted and Experimental Bubble Profile
(Temp = 387°C , Pressure = 15.51 kPa , Pole Defl = 20.6mm)

The neo-Hookean form of the strain energy function was also investigated for suitability in modelling the deformation process. For low values of extension ratios at the pole the predicted bubble height and the extension ratios were very similar to those given by the Mooney form.

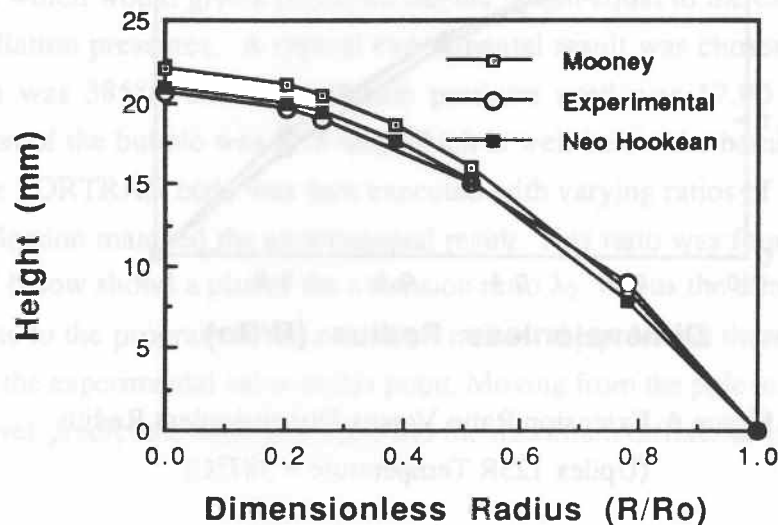


Figure 8. Predicted and Experimental Bubble Profiles
(Temp = 387°C , Pressure = 15.51 kPa , Pole Defl = 20.6mm)

From Figure 8 it can be seen that the neo-Hookean strain energy function gives a better fit to the experimental results for the bubble profile than the Mooney form. Figure 9 below shows the results for the circumferential extension ratio λ_2 . It can be seen that both numerical forms predict the same extension ratios. The constants C_{10} and C_{01} in the strain energy functions are

$$C_{01} = 0.76765301 \text{ N/mm}^2 \text{ and } C_{10} = 1.0929200 \text{ N/mm}^2 \quad \text{Mooney form and}$$

$$C_{10} = 2.2857322 \text{ N/mm}^2 \quad \text{neo-Hookean form.}$$

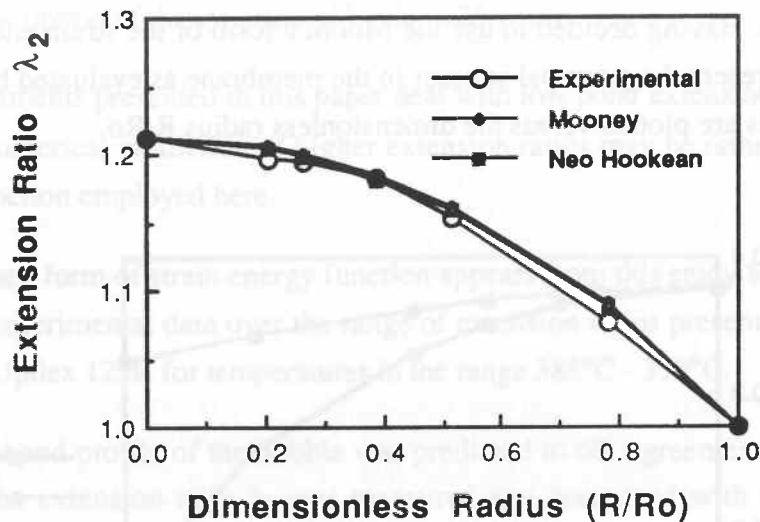


Figure 9. Experimental and Predicted Extension Ratios
(Temp = 387°C , Pressure = 15.51 kPa , Pole Defl = 20.6mm)

Both forms of strain energy were now compared to an experimental result with a greater bubble height. The bubble height chosen was 27.15mm and this corresponded to an inflation pressure of 24.83kPa at a temperature of 386°C. Figure 10 below shows the experimental and numerical results for this case. Again the ratio of the Mooney constants C_{10}/C_{01} was taken to be 0.697.

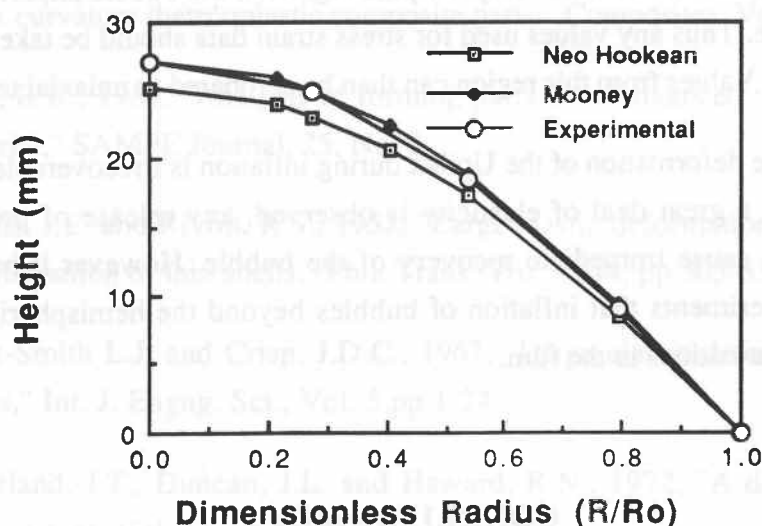


Figure 10. Experimental and Predicted Profiles for $\lambda_3 = 1.3199$
(Upilex 125R Temp. = 386°C Pressure = 24.83kPa)

The neo-Hookean form predicted a bubble height of 25.2mm while the Mooney form was more accurate with a bubble height of 27.1mm. The predicted extension ratios λ_2 were quite

similar for the two strain energy functions, however the Mooney form predicts more correctly the actual bubble profile and thus is considered a more suitable constitutive model for the Upilex 125R film. Having decided to use the Mooney form of the strain energy function it is now possible to present the principal stresses in the membrane as evaluated by the FORTRAN code. Both stresses are plotted versus the dimensionless radius R/R_0 .

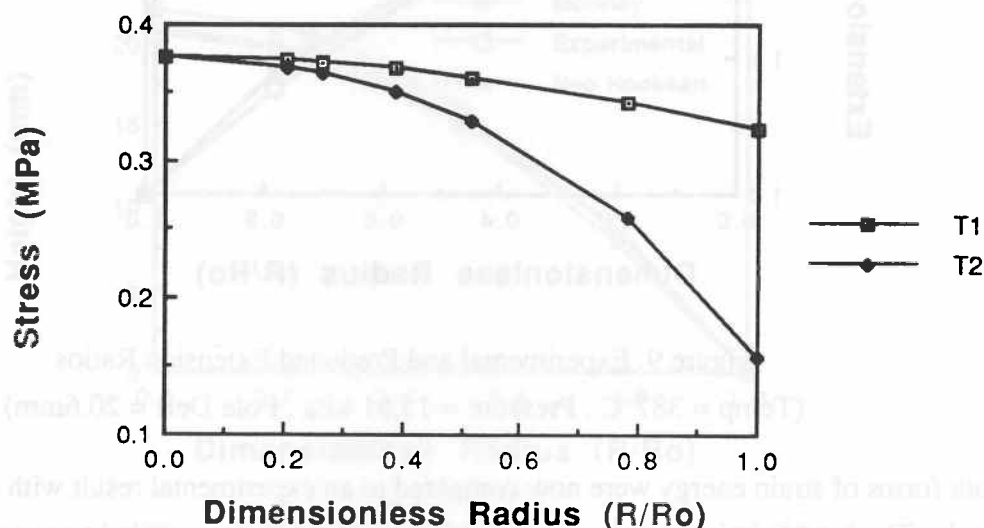


Figure 11. Plot of Principal Stresses in Upilex R film
(Upilex 125R Temp. = 386°C Pressure = 24.83kPa)

From Figure 11 it can be seen that only the pole region of the bubble experiences a true equal biaxial stress state. Thus any values used for stress strain data should be taken from this region of the membrane. Values from this region can then be compared to uniaxial test data.

While some of the deformation of the Upilex during inflation is irrecoverable during the actual inflation process a great deal of elasticity is observed, any release of pressure during the experiments will cause immediate recovery of the bubble. However it has been observed during some experiments that inflation of bubbles beyond the hemispherical height causes irrecoverable deformations in the film.

CONCLUSIONS

Biaxial inflation tests on Upilex 125R were successfully carried out at temperatures in the region of 380°C.

The large deformation theory developed for incompressible, isotropic elastic materials provides a good description of the inflation of Upilex 125R film.

The experiments presented in this paper deal with low polar extension ratios ie. $\lambda_{2p} \leq 1.5$, it is felt that numerical prediction of higher extension ratios may be rather difficult with the strain energy function employed here.

The Mooney form of strain energy function appears from this study to give the best correlation with the experimental data over the range of extension ratios presented, and can thus be used to model Upilex 125R for temperatures in the range 385°C - 390°C.

The height and profile of the bubble was predicted to 6% agreement of the large deformation theory. The extension ratio λ_2 was measured and compared with the Mooney constitutive relation prediction and the max difference was within 2.5%.

A true state of biaxial stress exists only in the polar region of the bubble, and the results from this area can be compared with uniaxial results to validate the constitutive relationship used.

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