Injection Moulding: Mathematical Modelling and Numerical Simulations

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Abstract

The considered process is the injection moulding of a polymeric melt in a porous pre-form of reinforcing elements. Process simulation is of considerable value in assessing production parameters and in improving the quality of manufactured products. The proposed model is deduced in the framework of deformable porous media. The model consists in a set of partial differential equations, time dependent, defining two coupled problems: mechanical equilibrium and diffusion in a permeable medium. The equations are solved by means of a finite element method.

1 Introduction

A number of composite materials are manufactured by an industrial process named resin transfer or injection moulding. It essentially consists in injecting a polymeric melt in the mould cavity. Before injection, a pre-form made of mats layers or aligned fibers is introduced in the cavity. After filling, the resin is cured (cross linked) by catalyzing agents, resulting in the formation of a thermosetting composite.

In several practical situations the pressure driving the flow is large enough to significantly compress the reinforcing network, especially ahead of the infiltration front (see, for example [1], [2] and [3]). It is fundamental that the matrix has filled back the whole die and the pre-form is not compressed in order to have an homogeneous product.

As a matter of fact, because of the high complexity of the phenomena occurring in the process, designing such moulds is a delicate operation that generally requires "trial-and-error" methods [4].

The final geometry of the cavity and the injection parameters are typically known only after many attempts, requiring the production of several moulds, as well as a lot of test and quality controls.

The aim of the mathematical modelling, which is carried out in the present paper, is to avoid this expensive and empirical stage by means of a model which allows to reproduce, through numerical simulations, the principal steps of the process.

The modeling of the filling stage remains a complex task. At this stage, in order to study the three-dimensional mechanical problem, we consider an isothermal process. We refer to [5] for one dimensional models which consider also a variable thermal field.

2 Modeling the infiltration process

The injection moulding process can be schematized as an infiltration of an incompressible liquid into a deformable porous medium composed of an incompressible solid. The spatial domain where the problem is formulated, defined as Ω, is the one occupied by the solid pre–form and in general depends on time, i.e. $\Omega = \Omega(t)$.

Since we are dealing with infiltration, Ω can divided into three regions: Ω^w, Ω^d and Ω^t which vary in time. The domains Ω^w and Ω^d correspond respectively to the region flooded by the infiltrating liquid (wet or saturated region) and the region not wet by the liquid (dry or unsaturated region) where the fluid permeating the porous solid is the air. The domain Ω^t represents the transitional layer between Ω^w and Ω^d. We assume that its thickness is very small if compared to the ones of the wet and dry regions.

The coupled flow deformation problem can be described in the framework of the theory of mixtures [6], [7] and [8]. The point is to characterize the unsaturated region. A simple way of doing that is to use the theory of solid-fluid mixtures assuming that the fluid component is itself a mixture composed by two fluids: the infiltrating liquid and air [9], [10] Chapter 2 and [11]. Denoting respectively by ϕ_f the volume fraction occupied by the mixture of two fluids we have

$$\phi_f = \phi_l + \phi_{air} \text{ where } \phi_l = s\,\phi_f \text{ and } \phi_{air} = (1-s)\,\phi_f\,, \tag{1}$$

being s a dimensionless parameter ranging between 0 and 1 called saturation, ϕ_l the volume fraction effectively occupied by the infiltrating liquid and ϕ_{air} the one occupied by the air. We remark that in the fully saturated region $s = 1$ (no air is present) while in the dry region $s = 0$ since there is no liquid. Obviously we must have

$$\phi_s + \phi_f = \phi_s + \phi_l + \phi_{air} = 1\,, \tag{2}$$

where ϕ_s is the solid volume fraction.

From now on the suffix s will indicate the solid, l the infiltrating liquid and f the fluid mixture.

Following the Eulerian formalism, in absence of chemical reactions and variable thermal field, neglecting capillary effects, gravitational force and inertial terms the equations for the three phases mixture are

$$
\begin{cases}
\dfrac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \vec{v}_s) = 0\,, \\[2ex]
\dfrac{\partial \left(s\, \phi_f\right)}{\partial t} + \nabla \cdot (s\, \phi_f\, \vec{v}_f) = 0\,, \\[2ex]
\nabla (s\, P) - \nabla \cdot \mathbf{T}' = 0\,, \\[2ex]
\vec{v}_l - \vec{v}_s = -\dfrac{\mathbf{K}_f\left(s, \mathbf{F}_s\right)}{\mu\, s \phi_f} \nabla P\,,
\end{cases}
\qquad \vec{x} \in \Omega\,,\ t > 0\,. \tag{3}
$$

where $(3)_1$ and $(3)_2$ are the mass balance equations, $(3)_3$ is the stress equilibrium equation and $(3)_4$ is Darcy's law. We remark that $\mathbf{K}_f$ is the permeability tensor depending on the deformation gradient $\mathbf{F}_s$ of the solid and on the saturation s. The constant μ is the liquid viscosity. The constitutive equations for the excess stress $\mathbf{T}'$ have still to be defined. In the present work we consider that the porous preform behaves elastically.

The assumptions considered to obtain (3) (see [12] for a critical discussion of its validity) restrict the analysis only to the infiltration stage. In fact, in the very first moments of contact between the liquid and the pre–form there is a rapid compression under the action of the pressure gradient. In particular, if the dry porous medium is assumed to behave as an elastic body, a shock wave propagates in the medium [13], [14]. During this transient, inertial terms play a dominant role, so they can not be neglected. Consequently, we focus on the instants following the transient. In our model, therefore, at $t = 0$ the dry solid is compressed under the full applied pressure, later, at $t > 0$, the infiltration front passes, it engulfs a slice of material which, relaxing, moves into the liquid.

We finally remark that the relationships between s and P are different for each porous solid and are usually determined by laboratory experiments (see, for instance, [10], [15] and [16]). Following [10], if we denote by P_{atm} the atmosperic pressure, we assume

$$
s = s\left(P\right)\,, \quad \text{with} \quad s\left(P\right) = 1 \ \text{ if } \ P > P_{atm}\,.
$$

3 Simulations

The case which has been considered is a cup and is represented in Fig. 1. ABAQUS computer code [17] has been used for solution of the system of equations governing the flow in deforming porous medium. The equations, written in a Lagrangian

i	γ_i $[KJ]$	α_i
1	297.8	25.0
2	2.513 10^{-2}	-9.07

Table 1: Values of the material parameters obtained form the curve reported in Fig. 2.

formulation, are solved by means of finite element method. For more details we refer to [18].

The pre-form material in the composite has a non linear elastic behavior. We have assimilated it to an elastomeric foam.

Figure 2 shows the stress-strain curve which has been considered. The plot refers to an uniaxial compression test. Besides, we have assumed that, in the non-stressed configuration $\phi_s = 0.14$. It is worth mentioning that large spreading of material properties are common among actual production, so that compression tests should be repeated for each new batch employed.

For an elastic porous foam the functional form of the energy is (see [19] for instance)

$$U = \sum_{i=1}^{N} \frac{2\gamma_i}{\alpha_i^2} \left[J^{-\frac{\alpha_i}{3}} \left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} \right) - 3 + \frac{1}{\beta_i} \left(J^{-\alpha_i \beta_i} - 1 \right) \right], \tag{4}$$

where γ_i, α_i, β_i, $i = 1 \ldots N$ are material dependent constants, $J = \det(\mathbf{F}_s)$ and λ_j^2, $j = 1, 2, 3$ is the j^{th} eigenvalue of the left Cauchy-Green tensor $\mathbf{B}_s$, i.e. λ_j represents the "stretch ratio" in the j^{th} principal direction of strain.

In the actual problem we have set $N = 2$. Material parameters have been identified by interpolating, using the least square method, the curve reported in Fig. 2. The identification of all the material parameters has been made possible by assuming a null Poisson coefficients η_i, $i = 1, 2$ which, for each term in the energy function are related to β_i by the expression

$$\eta_i = \frac{\beta_i}{1 + 2\beta_i} \quad i = 1, 2. \tag{5}$$

With these assumptions the values of γ_i and α_i are reported in Table 1.

The fluid polymer has been modeled as a non compressible, viscous, Newtonian fluid whose viscosity is $\mu = 1.6 \times 10^{-2}\ Pa \cdot s$.

The permeability tensor $\mathbf{K}_f$ has been assumed from literature [20] and [21], i.e.

$$(\mathbf{K})_{ij} = s^3\, K_o\, 10^{-a\,\phi_s}\, \delta_{ij}, \tag{6}$$

where $K_o = 1,473 \times 10^{-10}\ m^2$ and $a = 6.965$.

The saturation law, which we have assumed, is logarithmic

$$P = \frac{1}{A} \ln \left[\frac{s - s_o}{(1 - s_o) + B\,(1 - s_o)} \right]. \tag{7}$$

The term s_o is the minimum saturation value acceptable: in our simulations it has been assumed 0,02. In this way wet and dry zone are separate by a partially saturated region whose width depends by the parameters of the saturation law, in our examples $A = 0.74\ KPa^{-1}$ and $B = 50$.

Boundary conditions are applied to the die in order to constrain any rigid body displacement or rotation. There are no degrees of freedom constrained among the nodes of the matrix.

Two steps constitute the load history. First the fluid is admitted inside the die and it compress the matrix material. The second step is the absorption of the fluid by the porous material.

In the case of the cup the liquid is injected into the pre–form from the upper side. The applied pressure is $30\,KPa$. Figure 3 shows the void ratio, defined as $(1 - \phi_s)/\phi_s$, after the compression phase. The void ratio in the undeformed configuration is 6.14. After the compression (i.e. at the end of the first step) the void ratio is uniform in the cylindrical part but not in the spherical-one. The biggest inhomogeneities are in correspondence to the connection between the two parts.

The infiltration process ends in about 25 seconds but the time required by the porous solid to recover the configuration it had before the compression is about 40 seconds. In Fig.s 4 (a) and (b) the saturation at 0.5 second and at 5 second is reported. It is possible to observe that at $t = 0.5\ s$ the infiltration front is very steep, although at 5 second it has been widened due to the smoothing of the pressure field. The saturation in the dry region is, also in this case, slightly bigger than 0.02.

The main technological result is the possibility to discriminate between the instant at which the infiltration front has reached the bottom of the die and the one corresponding to uniform fluid pressure all over the part. In a practical application, the first instant is revealed from the fluid pouring out of the outlet gates. Nevertheless the process is terminated only when the fluid pressure has become uniform and the pre-form has recovered the initial shape.

The correct prediction of die filling and of injection time constitutes a serious difficulty to the expansion of the injection molding practice for composite materials. Design methods commonly employed for the injection of polymer cannot be transferred in a straight way to composites. The behavior of the mixture of molten resin and pre–form has to be described correctly in order to predict the feasibility of the process. In the present paper we propose a model of infiltration in deformable porous medium which is well apt to numerical simulations.

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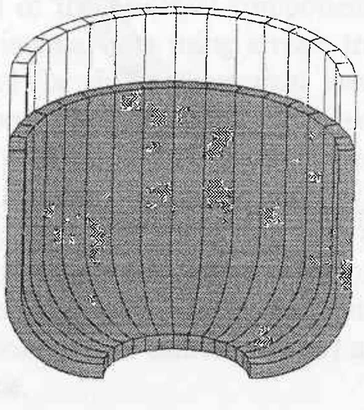


Figure 1: Deformed and undeformed geometry of the component in the first case study. One half of the part is represented. The external radius of the cup is 22mm large, 2mm thick and 42mm high, the radius of the central hole is 9mm. The fluid is injected into the cup through the upper side.

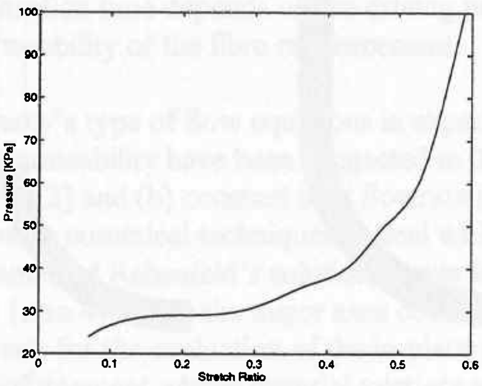


Figure 2: Relationship between stress and stretch ratio for an uniaxial compression test.

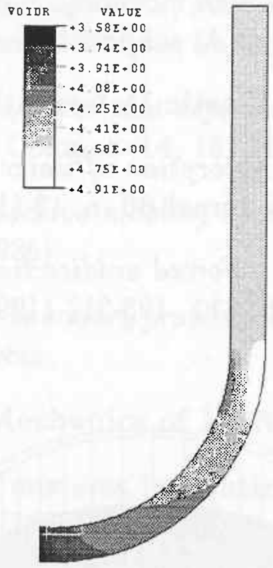
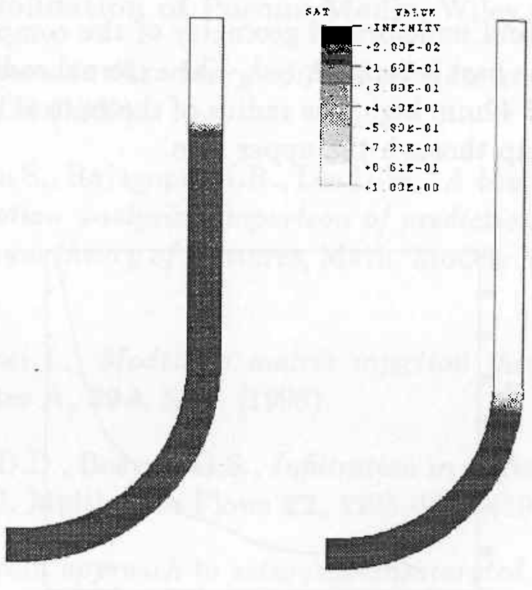


Figure 3: Void ratio of the the solid preform after the intial compression.



(a) (b)

Figure 4: Saturation at t=0.5 s (a) and at t=5 s (b).