A VISCOELASTICITY MODEL FOR DEFORMABLE REINFORCEMENT MATERIAL IN LIQUID COMPOSITE MOULDING

J. Bréard*[1], Y. Henzel[1] and F. Trochu[1]

S. Cadinot[2], A. Saouab[2] and G. Bouquet[2]

[1] Centre for Applied Research on Polymers
Mechanical Engineering Department, Ecole Polytechnique of Montréal
P. O. BOX 6079, Station "Centre Ville", Montréal, Québec, H3C 3A7, Canada.

[2] Université du Havre, Laboratoire de Mécanique
Quai Frissard, BP 265, 76055 Le Havre, France

ABSTRACT

In this paper, we are going to concentrate on the injection stage of the LCM process (Liquid Composite Moulding), in fact on the flow through a fibrous structure. Before to make a numerical simulation, it is necessary to present the approach to describe the flow displacement through a saturated porous medium. We introduced the problem of the reinforcement's deformation in order to give the necessary elements for the interpretation of the mechanism of flow propagation through a porous medium. So, the medium will be considered as saturated, deformable, porous and the traditional tools of mechanics of continuous media will be used to describe the behaviour of this solid. We give a first approach relating to the local variation of the porosity problem from a thermodynamical analysis. The constitutive law of the viscoelasticity medium is obtained and is integrated in a global approach that allows us to explain the coupling between the fluid and the skeleton of the reinforcement material. Examples of experimental studies are presented introducing a unidimensional configuration in porous media. An experimental study allows us to analyse the local porosity variation's behaviour.

KEYWORDS: Porous Media, Viscoelasticity, Porosity, Deformable, Physical Analysis

INTRODUCTION

For about thirteen years, LCM (Liquid Composite Moulding) processes have largely developed both on the technical aspects and on physical phenomena modelling. The general principal of these processes consists in injecting a fluid (usually resin) into a fibre reinforcement that constitutes a porous medium. Hence the study of resin flow can be generalised to flow in porous media. Various couplings of phenomena appeared among rheological, thermal, chemical and elastic properties. Lately we realised that elastic behaviours had to be clarified since new processes such as CRTM and VARI become more

* Author to whom correspondence should be addressed, Email: breard@meca.polymtl.ca

common. Hence one needs a deeper understanding of characteristics of compressibility and relaxation of fibrous structures.

The need for efficient simulation softwares is increasingly important for industrial applications. The numerical programming of these softwares uses very simple physical laws. The porous medium is indeed considered as completely saturated, deformable yet the elasticity laws are quite simplified. In the literature, only empirical or semi-empirical models are found that do not allow any link with the underlying physical behaviours. In order to reach a proper analysis of the later, one must take deformation problems into account for both dry and saturated reinforcements. This works provides with preliminary considerations about the modelling of these phenomena.

1. FLOW THROUGH A DEFORMABLE POROUS MEDIUM

Before developing the behaviour law that will be used in our modelling, it is important to go back on the various laws governing fluid-structure interaction in our problem. In this paper, solely the isothermal part of this development shall be presented.

To do so, Biot's [1] theory will be presented. We now consider the study of porous media saturated by a fluid. Saturation aspects will not be taken into account. On this aspect, works are being validated and were recently published by Henzel [2]. Unsaturated media only differ by the introduction of a third phase (air bubbles) in the formulation of flow in deformable porous media.

For this kind of a problem two approaches can be carried out. The first one uses homogenisation procedures that allow to go to macroscopic laws from microscopic considerations. The development of these methods can be found in the work of Suquet [3]. The second approach consists in assuming the concepts and principles of continuum mechanics are applicable to macroscopic and measurable entities. This is an older approach and was developed by Biot [4]. We chose to go along that way for this is a more physical description. The main hypotheses will be presented concentrating on the expressions adapted to our application as presented in Bréard [5]. Before going any further, measurable quantities will be presented as shown on Fig. 1.

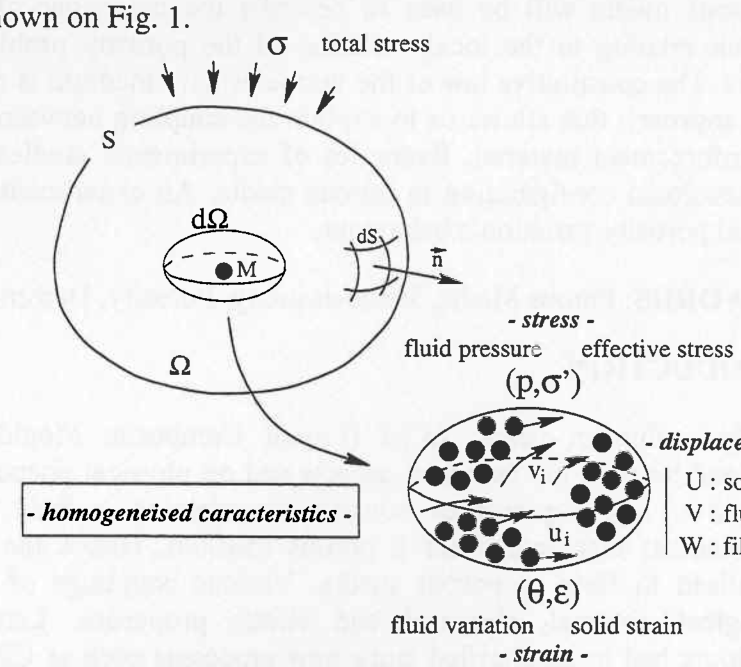

Fig. 1: Modelling of the homogenised flow

We recall that motion description can be based on two different approaches. In the Lagrangian representation, one focuses on the history of a medium's region with a fixed identity. Whereas, in the Euler representation, the medium's characteristics at a point are rather considered. Movement mathematical laws formulation is often easier with Lagrange variables. Yet, the second type of description has been used because the resolution of this practical problem is usually easier with Euler's formalism. In this study, at the microscopic scale, a part of the medium is the locus of various displacement fields (Fig. 1) : u_i the preform's displacement field (if it is deformable) and v_i the fluid's displacement field.

Consider a spherical elementary volume centred at M. φ is defined as the ratio of the fluid volume and the total sphere volume. If $\varphi = 1$, the sphere only contains fluid and only solid for $\varphi = 0$. For a set of spheres, which radius rises up from the same centre point M, φ statistically tends towards a finite value that can be considered as reached for a radius r_0. This ϕ value is called structure's porosity, around point M. The volume corresponding to r_0 is called Representative Elementary Volume (REV).

$$\phi = 1 - TvF = \frac{Vol_v}{Vol_t} \tag{1}$$

with TvF the volumic fibres rate, Vol_v the void volume (pores), Vol_t total REV volume. Bear [6] defines different domains according to r_0. A porous medium will have a homogeneous porosity distribution if ϕ has the same value on every point of the medium. Also, the porosity being the surfacic rate of fluid taken on a disc with radius r greater than r_0, is independent from the orientation of the disc centred on point M. r_0 is the spatial scale allowing to build, in the domain, a continuous medium model : considering an elementary volume around point M, with a radius smaller than r_0, the scale is microscopic, and the material heterogeneous. Conversely, for radius greater than r_0, we are in a macroscopic homogeneous scale. We will hence assume that continuum mechanic principles and results apply. In both cases, r_0 is much smaller than a characteristic dimension of the studied system.

To complement this model, a domain $\Omega_{REV}(M)$, centred on M, whose characteristic scale is greater or equal to r_0 will be considered. The saturated impregnation at M notion is extended to all the pores included in Ω_{REV}. On the $\Omega_{REV}(M)$ domain, three macroscopic displacement fields will be defined. $U(M,t)$: the displacement vector macroscopic mean in the solid part of Ω_{REV}, $V(M,t)$: displacement vector macroscopic mean in the fluid part of Ω_{REV} (V is prolonged by the 0 value in the non-impregnated domain)and $W(M,t)$: filtration vector mean in Ω_{REV} or relative displacement vector of the filtrated fluid part of Ω_{REV} versus the solid part, where W is defined by the relation $W_i(M,t) = V_i(M,t) - U_i(M,t)$. Similarly, macroscopic stress and strain are respectively defined by (σ'_{ij}, p) and $(\varepsilon_{ij}, \theta)$. We will come back on the definition of these later on. These physical quantities, denoted by a capital letter, e.g. X, are defined as macroscopic values over the domain Ω_{REV} such as

$$X(M,t) = \frac{1}{\Omega_{REV}} \int_{\Omega_{REV}} x(M,t)\, d\Omega_{REV} \tag{2}$$

In the sequel, we shall only work on the REV scale of this continuous model. As two states (phases) coexist in this space, we talk about a biphasic model. There is a solid skeleton saturated by a fluid phase. The mechanical properties of such a structure depend on intrinsic skeleton and fluid properties and on the interactions between those phases.

The porous medium was defined as a continuous biphasic medium. Under the influence of external forces or/and of pressure gradients of the saturating fluid, the porous medium will change its shape. The observable deformation is, in fact, the skeleton's (ε_{ij}) and this is the one that should be described. This description is absolutely identical to those carried out on a classical continuous medium.

With the small displacement hypothesis for the solid part, only first order terms will be retained. Note that this does not apply to the fluid phase. Notwithstanding, it is sufficient to consider slow movements for which Darcy's Law [7] is suitable to describe the filtration displacement through the skeleton.

Fluid volume variation

From the fluid flow rate through S and with Green's Theorem we get the following equation:

$$\int_S W_i n_i \phi dS = \int_\Omega (\phi W_i)_{,i} d\Omega \tag{3}$$

The quantity $(\phi W_i)_{,i}$ then corresponds to a relative flow rate towards the exterior of Ω a time t. Hence we define

$$\theta(M,t) = -(\phi W_i)_{,i} \tag{4}$$

as the fluid volume variation around a given point M at time t. Filtration displacements, given by the field W, are defined using θ.

Mass conservation

Since, in our case, both solid and fluid phases are uncompressible, the corresponding mass conservation equations can be written as follows:

$$\begin{cases} fluid: & \dot{\phi} + (\phi \dot{V}_i)_{,i} = 0 \\ solid: & -\dot{\phi} + ((1-\phi)\dot{U}_i)_{,i} = 0 \end{cases} \tag{5}$$

The continuity equation for the whole domain is defined from the relative fluid displacement W and equations (5), leading to:

$$(\phi \dot{W}_i)_{,i} = -\dot{\theta} = -\dot{\varepsilon}_{kk} \tag{6}$$

where ε_{kk} is the trace of the skeleton strain tensor that can depend on the saturation of the medium. This saturation phenomenon can also be taken into account in the continuity equation. In unsaturated flows the porosity variation term associated to fibre rearrangement within the fabric is neglectible regarding the first term involving the saturation degree variation. Now, this phenomenon will not be considered and we will assume that the medium is fully saturated.

Motion quantity conservation

The flow goes towards the exterior through the surface S of a medium containing a fluid phase ϕS and a solid phase $(1-\phi)S$. As the filtration vector was defined, the considered filtration velocity is hence $\dot{W}$, namely the Lagrangian flow. The flow through the surface can hence be expressed as the filtration velocity vector (Darcy's velocity : q) :

$$q_i = \phi \dot{W}_i \tag{7}$$

So, flow through porous media is described by Darcy's Law:

$$q_i = \phi \dot{W}_i = -\frac{K_{ij}}{\mu} p_{,j} \tag{8}$$

Where q_i is the average velocity, μ the viscosity and $p_{,j}$ the pressure gradient of the fluid. It is valid in permanent regime. Unsaturated flows are usually considered as quasi-stationary phenomena, a succession of stationary states.

2. MODELLING THE HOMOGENISED PROBLEM

In soil mechanics, with the full saturation hypothesis, Terzaghi's Law [8] is frequently used. It consists in decomposing macroscopic total stress σ_{ij} into effective stress σ'_{ij} that act on the solid skeleton and hydrostatic stress $(-p\delta_{ij})$ acting on the fluid phase (Fig. 1), with the relation:

$$\sigma_{ij} = \sigma'_{ij} - p\delta_{ij} \tag{9}$$

Terzaghi's law is more restrictive that Biot's theory. Indeed, in Terzaghi, this leads to decoupling between fluid and solid behaviours. To evaluate effective stress, the medium is assumed to be fully constituted of the solid phase and then fluid pressure effects are subtracted. The fluid behaviour considers only its volumic dilatation which corresponds to the separation mentioned beforehand. It should be brought up here that the strain tensor ε_{ij} is related to the average or macroscopic displacement of the solid-fluid ensemble of the REV. This tensor is not related to the only solid phase. Furthermore, Biot's theory solely applies to small displacements and ε is considered small as well as the fluid displacements.

Terzaghi's law, compared to Biot's theory, assumes that the fluid action can be considered as an exterior force acting on the solid skeleton. The law (Eq. (9)) injected into the equilibrium relation leads indeed to

$$\sigma'_{ij,j} - p_{,j} = 0 \tag{10}$$

The action of the fluid on the solid part being transduced by fictitious volumic forces of intensity $(-p_{,j})$ Hence, under the small strain hypothesis for both solid and fluid components, Terzaghi's law is valid in a more restrictive hypotheses context than Biot's theory. However, it can be very useful in some flow conditions. The decoupling hypothesis allows indeed to study large fluid movements independently of small strains of the skeleton. To do so, the fluid pressure field is determined with Darcy's law and the fluid mass conservation law. Integrating this pressure field into Eq. (5) one obtains the equilibrium equations for the solid phase whose behaviour law can be chosen as suitable. In our case, we will consider the consolidation model as in Biot [4]. The coupling will be given by the following relations:

$$\begin{cases} \sigma'_{ij,j}(U) = p_{,j} \\ \left(K_{ij} p_{,j}\right)_{,i} = -\dot{\varepsilon}_{kk}(U) \end{cases} \tag{11}$$

σ'_{ij} corresponding to the material's behaviour law (with $\sigma_{ij} = \sigma'_{ij} - p\delta_{ij}$) and $\varepsilon_{ij} = \frac{1}{2}\left(U_{i,j} + U_{j,i}\right)$. The unknowns U and p are functions of M and t, M being a point within the bounded domain Ω. The boundary conditions will depend on each individual problem and could be among $p|_{\Gamma} = p_{\Gamma}$, $K_{ij} p_{,j} \cdot n_j|_{\Gamma} = Q_{\Gamma}$, $U|_{\Gamma} = U_{\Gamma}$ and $\sigma_{ij} \cdot n_j|_{\Gamma} = \sigma_{\Gamma}$, Γ being a portion of the boundary surface S decomposed into complementary portions. Some authors like Auriault [9], Coussy [10] have discussed about the resolution of such equations.

3. A VISCO-ELASTICITY MODEL FOR POROUS MEDIA

The aim of this paper is not to give a complete resolution scheme of such a problem but to propose a behaviour law well adapted and physically based. The cause of dissipative mechanisms observed experimental studies must be looked into absolute local movements of both fluid and solid phases. We will try to model this phenomenon introducing viscoelastic models. Indeed, many authors as Cai [11], Kim [12] and Gauvin [13] proposed models for fibrous preforms compaction and relaxation. However, these are often linked to empirical or semi-empirical considerations.

The whole approach will not be presented, but only the global shape of the proposed model. It will be defined as follows, according to uni-dimensional compaction (constant speed $\dot{\varepsilon}_0$) and relaxation (imposed strain ε_0) experiments. The medium is still considered as homogeneous. Our modelling concentrates on the average dissipation in the REV from a macroscopic point of view, without separating both phases contributions. The displacement is hence a macroscopic average on the fluid-solid ensemble as introduced previously. The origins of this dissipation are many, besides Biot-like phenomena. Most likely mechanisms are linked to capillary forces, friction between fibres, local fluid movements,... It is useless for us to consider all of these effects. This is why only the simpler models will be considered, in particular linear viscoelastic models. They are well-adapted to fast imposed stress variations such as a flow rate step or a preform compaction. These models need the knowledge not only of present stress and strain values but also of their history. These are memory models. Hence, it is essential to study their delayed behaviour, e.g. the time effects such as those linked to strain velocity ($\dot{\varepsilon}_0$). This model is only valid for a unidimensional behaviour. It will be generalised later on but readers may already refer themselves to works of Salençon [14] and Lemaitre [15]. Rheological models for viscous dissipative phenomena have been used as a first basis. They will be represented by the following arrangement in Fig. 2.