CREEP AND RECOVERY IN BI-POLYMER COMPOSITES

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Abstract Two types of spring-dashpot models are applied to the creep observed from two dissimilar polymers connected in parallel to share an axial tensile load. An initial load is applied for 60 h. Then, following a 24 h period of recovery, the load is increased and sustained for a further 60 h. These interchanging periods of recovery and creep are repeated until the composite loses its load bearing capacity. A continuous record of displacement versus time is taken for the test duration. The forward creep strains show better agreement with the Burger model than the standard linear solid (SLS). The Boltzman superposition principle allows the recovered strain, arising from load removal, to be predicted within each model. Overlaying the predicted and experimental curves reveals how the theoretical errors accumulate with time. The greatest error was found from combining nylon with polypropylene, where the theory underestimated the 7.5% strain observed by 35% under a nominal stress of 17.5 MPa. The error was less for five further parallel polymer combinations at comparable stress levels. The model relies upon pre-requisite knowledge of each individual creep response to a similar load-unload sequence.

INTRODUCTION

Polymers are renowned for their dimensional instability when required to bear loads at moderate temperatures. Low and high density polyethylene each show considerable creep strain even under low loads when compared to nylon and polypropylene [1]. Layers of these polymers, with contrasting creep resistances may be used to produce a simple composite that shares the load with an improved creep resistance. This paper examines the appropriateness of rheological models to predict tensile creep and recovery in the composite. Various combinations of springs and dashpots have been used to describe creep and recovery of viscoelastic solids [2]. The standard linear solid is the most appropriate to begin this study though an additional dashpot element in Burger's model [3] was found to be necessary for improving the predicted creep response to the pattern of loading employed. The superposition principle accounts for recovered strain within each model during the interspersed periods of load removal. The strategy for predicting creep was to take two SLS models in parallel to support a given load. Imposing the conditions of equilibrium and compatibility leads to a differential equation that can be solved for creep and recovery. While this method may be extended to multi-layered composites, in two or more materials, the basic bipolymer serves to reveal the limitations of the model chosen. For example, the SLS shows that the strain attains an asymptotic value after a certain time, this being consistent with creep observed under lower loads. The Burger model reveals a steady creep rate at a comparable time and this was found to be consistent with creep under higher loads. With increased loading the observed rate was found to increase continuously in an unpredictable manner as a neck formed. At this point the material was deemed to have failed.

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THEORETICAL

The standard linear solid (Fig. 1a) consists of a spring with stiffness E_1 connected in series to a parallel Voigt combination of a second spring with stiffness E_2 and a dashpot with viscosity μ.

Figure 1 Spring-dashpot combinations within (a) SLS and (b) Burger

The creep and recovery responses of an SLS to a stress input σ is found from (i) identical stresses in the spring and Voigt element and (ii) that the total strain is the sum of the two components:

$$\sigma = E_1\varepsilon_1 = E_2\,\varepsilon_v + \mathrm{d}\varepsilon_v/\mathrm{d}t \tag{1}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_v = \sigma/E_1 + \varepsilon_v \tag{2}$$

Eliminating ε_v between eqs(1) and (2) leads to the governing differential equation:

$$\mathrm{d}\varepsilon/\mathrm{d}t + (E_2/\mu)\varepsilon = (\sigma/\mu)(1+E_2/E_1) + (1/E_1)\mathrm{d}\sigma/\mathrm{d}\,t \tag{3}$$

For creep assume that $\sigma = \sigma_o$ = constant for all t in eq(3):

$$\mathrm{d}\varepsilon/\mathrm{d}t + (E_2/\mu)\varepsilon = (\sigma_o/\mu)(1 + E_2/E_1) \tag{4}$$

The solution to eq(4) gives the time dependent creep strain response:

$$\varepsilon = \sigma_o/E_1 + (\sigma_o/E_2)[1 - \exp(-E_2 t/\mu)] \tag{5}$$

For recovery we set $\sigma_o = 0$ at time $t = t_1$. An instantaneous elastic strain σ_o/E_1 is recovered and thereafter, for a time $t' = t - t_1$, eq(3) yields:

$$\mathrm{d}\varepsilon/\mathrm{d}t' + (E_2/\mu)\varepsilon = 0 \tag{6}$$

For $t' > 0$, the time dependent recovered strain follows from the solution to eq(6) noting that eq(5) supplies the creep strain for time $t > t_1$ as:

$$\varepsilon = (\sigma_o/E_2)\exp(-E_2 t/\mu)[\exp(E_2 t_1/\mu) - 1] \tag{7}$$

Eqs(5) and (7) give respective asymptotes to the creep and recovered strain of $\sigma_o/E_1 + \sigma_o/E_2$ and

zero at infinite times. This may not be representative where a constant rate of straining prevails during creep. Now let us assume that two polymers A and B, in a parallel combination, can be modelled with two separate SLS's connected in parallel as shown in Fig. 2.

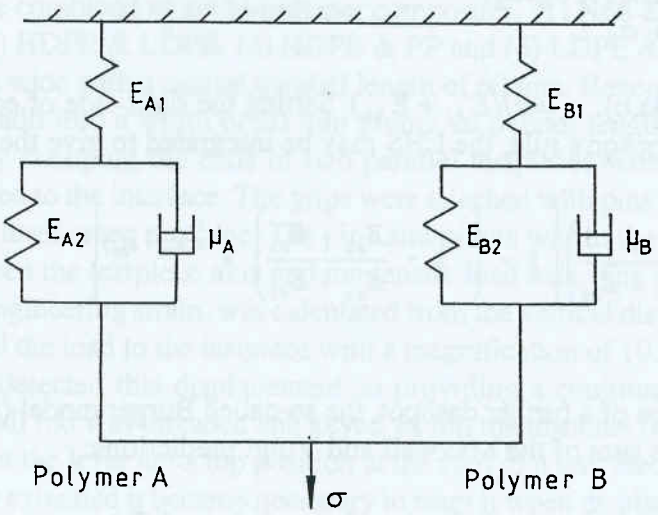

Figure 2 A bi-polymer SLS model

Thus with two SLS's, side-by-side, the equilibrium condition becomes:

$$\sigma = \sigma_A + \sigma_B = E_{A1}\varepsilon_{A1} + E_{B1}\varepsilon_{B1} = [E_{A2}\varepsilon_{AV} + \mu_A(d\varepsilon_{AV}/dt)] + [E_{B2}\varepsilon_{BV} + \mu_B(d\varepsilon_{BV}/dt)] \quad (8)$$

where subscript v denotes the Voigt parallel spring-dashpot combination. The first compatibility condition is that each polymer suffers the same total strain $\varepsilon = \varepsilon_A = \varepsilon_B$ where:

$$\varepsilon_A = \varepsilon_{AV} + \varepsilon_{A1} \Rightarrow \varepsilon_{AV} = \varepsilon - \sigma_A/E_{A1} \quad (9a)$$
$$\varepsilon_B = \varepsilon_{BV} + \varepsilon_{B1} \Rightarrow \varepsilon_{BV} = \varepsilon - \sigma_B/E_{B1} \quad (9b)$$

The second compatibility condition is that the total strain rates are the same in SLS A and B. Differentiating eq(5) gives:

$$d\varepsilon/dt = (\sigma_A/\mu_A)\exp(-E_{A2}\,t/\mu_A) = (\sigma_B/\mu_B)\exp(-E_{B2}\,t/\mu_B) \quad (10a,b)$$

From eqs(10a,b), the stresses in each polymer are:

$$\sigma_A = \mu_A \exp(E_{A2}t/\mu_A) \times d\varepsilon/dt \quad (11a)$$
$$\sigma_B = \mu_B \exp(E_{B2}t/\mu_B) \times d\varepsilon/dt \quad (11b)$$

Substituting eqs(9a,b) and (11a,b) into eq(8) and, to a first approximation, neglecting the stress rates $d\sigma_A/dt = d\sigma_B/dt = 0$ under a constant stress input leads to:

$$\int_{\varepsilon_o}^{\varepsilon} \frac{d\varepsilon}{\sigma - \varepsilon(E_{A2} + E_{B2})} = \int_0^t \frac{dt}{\mu_A[1-(E_{A2}/E_{A1})e^{E_{A2}t/\mu_A}] + \mu_B[1-(E_{B2}/E_{B1})e^{E_{B2}t/\mu_B}]} \quad (12)$$

where the lower strain limit is the instantaneous elastic strain response to σ within springs A_1

and B_1. Since their ends are fixed and the load is shared the conditions of equilibrium and compatibility apply:

$$\varepsilon_o = \sigma_A / E_{A1} = \sigma_B / E_{B1} \tag{13a}$$

$$\sigma = \sigma_A + \sigma_B \tag{13b}$$

and from eqs(13a,b): $\varepsilon_o = \sigma/(E_{A1} + E_{B1})$. Setting the right-side of eq(12) to I, a numerical value obtained by Simpson's rule, the LHS may be integrated to give the creep strain as:

$$\varepsilon = \left(\frac{\sigma}{E_{A2} + E_{B2}}\right)\left[1 - \left(1 - \frac{E_{A2} + E_{B2}}{E_{A1} + E_{B1}}\right) e^{-I(E_{A2} + E_{B2})}\right] \tag{14}$$

With the addition of a further dashpot, the so-called Burger model (see Fig. 1b) supplies a creep strain that is the sum of the Maxwell and Voigt predictions:

$$\varepsilon = (\sigma_o / E_2)[1 - \exp(-E_2 t / \mu_2)] + \sigma_o (t/\mu_1 + 1/E_1) \tag{15}$$

Equation (15) shows a steady creep rate asymptote $\dot{\varepsilon} = \sigma_o/\mu_1$ for $t \to \infty$. Within the Burger model only the Voigt element recovers strain and so eq(7) again applies with $\mu = \mu_2$. For the parallel bi-polymer (see Fig. 3) the Burger creep equation is identical in form to eq(14) but with I redefined as:

$$I = \int_0^t \frac{dt}{\mu_{A2} - \dfrac{\mu_{A1}(E_{A2}/E_{A1} + E_{A2}t/\mu_{A1} + \mu_{A2}/\mu_{A1})^{-1}}{[1 + (\mu_{A1}/\mu_{A2})\, e^{-E_{A2}t/\mu_{A2}}]} + \mu_{B2} - \dfrac{\mu_{B1}(E_{B2}/E_{B1} + E_{B2}t/\mu_{B1} + \mu_{B2}/\mu_{B1})^{-1}}{[1 + (\mu_{B1}/\mu_{B2})\, e^{-E_{B2}t/\mu_{B2}}]}} \tag{16}$$

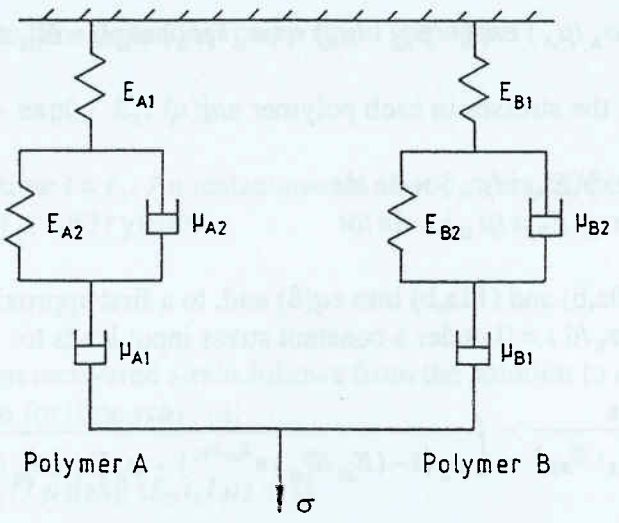

Figure 3 A bi-polymer Burger model

EXPERIMENTAL

Nylon (N66), high density polyethylene (HDPE), low density polyethylene (LDPE) and polypropylene (PP) were combined as six bi-polymer composites: (1) N66 & HDPE, (2) N66 & LDPE, (3) N66 & PP (4) HDPE & LDPE, (5) HDPE & PP and (6) LDPE & PP. Each testpiece was 4 mm thick, 10 mm wide with a central parallel length of 60 mm. Beyond this each end was enlarged with 60 mm radii into a width of 20 mm giving an overall length of 150 mm. A bi-polymer was formed by clamping the ends of two parallel testpieces with serrated grips. No bonding agent was applied to the interface. The grips were attached with pins to the straining bars and shackles of a single lever creep machine. The pins and pivots within the load chain ensured vertical alignment between the testpiece axis and the tensile load axis. The displacement of the testpiece and hence its engineering strain, was calculated from the vertical displacement of a rigid pivoted lever that carried the load to the testpiece with a magnification of 10. A long stroke 100 mm LVDT transducer detected this displacement so providing a continuous chart recording during test. The lower pull rod was threaded and keyed within the machine body. This was fitted with a handwheel to reset the lever at its top position at the start of a test. Because the end of the lever fell as the testpiece extended it became necessary to reset it when displacements were large, i.e., with LDPE present. With the load sequence employed the chart displayed alternating periods of creep and recovery. The plots were regenerated within a compressed time scale for the purpose of overlaying theoretical predictions. To obtain the model constants the individual polymers were subjected to a similar loading-unloading pattern. The strains at times 0, 10 and 20 h were taken for the determination of the 3 SLS constants. An additional time of 40 h was used for obtaining the fourth Burger constant. Constants were found for each creep curve and then averaged to yield the material constants given in Table 1. Typical predictions to single polymer test data are shown in Figs 4 and 5. When based upon these average constants, eqs(5) and (15) are seen to reproduce the experimental curves with acceptable accuracy. A trend appears, however, when applying this approach to polyethylene, where both models overestimate creep strain at low loads and underestimate it at high loads. Polypropylene displays a similar smaller discrepancy but for nylon the trend is reversed. Neither model is able to account for the irregularities in creep curves due to temperature fluctuations within the 60 h time span. The assumption of constant stress as a source of error is also recognised.

RESULTS AND DISCUSSION

In applying eqs(14) and (16) the constants in Table 1 were found from curve fitting and thereby are not true material constants. The integrals I were evaluated at a given time by repeatedly applying Simpson's rule. Boltzmann's superposition principle [4] predicts that the recovery curve is simply an inversion of the previous creep curve when all the load is removed. Since less time was allowed for recovery the residual strain was added to the theoretical creep strain arising from the re-application of an increased load. In this manner the predicted strain-time response to repeated loading and unloading was found. Typical comparisons with the experimental curves for four bi-polymer combinations are shown in Figs 6 - 9. The broad conclusion that may be drawn from this is that Burger's model provides a steady creep rate asymptote while the SLS reveals a constant strain asymptote. Clearly, the former is more descriptive of the observed behaviour for the times allowed. Discrepancies in the strain magnitudes arise from assuming a constant stress. Whilst the load is constant the stress will increase with decreasing section area, particularly at high loads where both models underestimate creep strain. At lower loads the

models reproduce the trends observed more faithfully but there remains the need to employ true stress and real material constants for higher loads and temperatures within the derivations.

Table I Rheological Model Constants

	Standard Linear Solid			Burger - 4 Component Model			
Polymer	E_1 (MPa)	E_2 (MPa)	μ_1 (MPah)	E_1 (MPa)	E_2 (MPa)	μ_1 (MPah)	μ_2 (MPah)
N 66	3.72	38.82	253.21	3.72	45.04	11178	227.4
HDPE	1.64	4.62	22.62	1.64	5.2	1483	20
LDPE	0.85	3.51	16.77	0.85	4.24	485.2	11.74
PP	5	11.25	67.44	5	13.71	2053.7	58.1

The following theoretical modification accounts for a time-varying stress. The true stress σ follows from the nominal stress σ_0 when incompressibility is assumed as:

$$\sigma = \sigma_0 \, e^{\varepsilon_n} \tag{17}$$

in which the natutal (logarithmic) strain is: $\varepsilon_n = \ln(1 + \varepsilon)$ where ε is the engineering strain used here. Equation (17) permits an account of the stress rates within eq(3). Differentiating eq(17):

$$d\sigma/dt = \sigma \, d\varepsilon_n/dt \tag{18}$$

A creep equation, embodying eqs(17) and (18), should fit single and bi-polymer behaviour more reliably. For example, it is anticipated that in applying such modifications the SLS may overcome its restrictive constant creep strain asymptote for extend times.

CONCLUSIONS

Creep under repeated loading-unloading is better predicted by the Burger model. When the recovery phase is estimated by the superposition principle it will depend upon the forward creep strain prediction. In contrast, the basic models employed do not differ in their predictions to the recovered strain (i.e. eq(7) applies to both the SLS and Burger models). The accuracy of a rheological model depends upon its inputs. Material constants become inaccurate under a time varying stress and therefore could be determined more reliably under stress and strain rate controlled tensile testing. Environmental influences such as changing temperature and humidity are not explicit variables within these models but might be accounted for within the model constants, i.e. where these influences upon the material constants are known.

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(3) Brown, S.F. *Creep of Engineering Materials* (ed Pomeroy,C.D.) , MEP, 1978, p.215.
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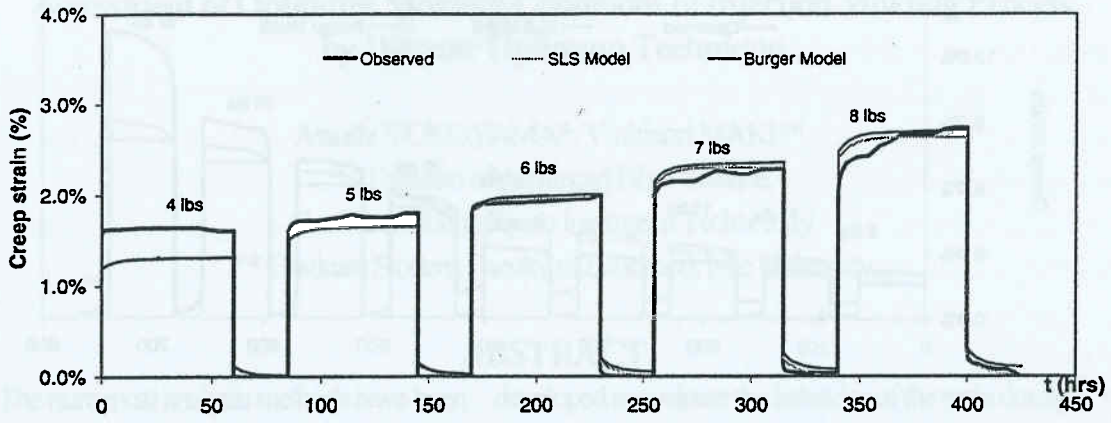

Figure 4. Observed and predicted creep and recovery behaviour of nylon 66

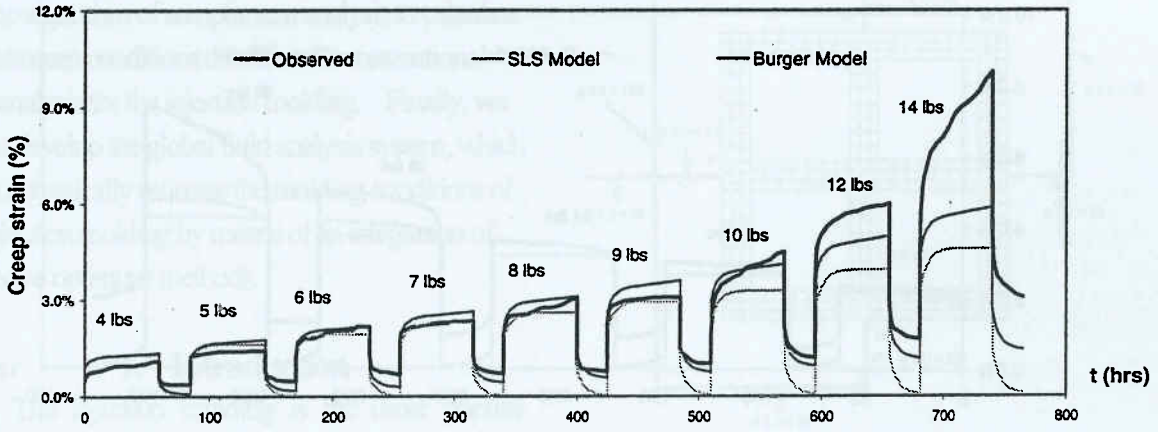

Figure 5. Observed and predicted creep and recovery behaviour of polypropylene

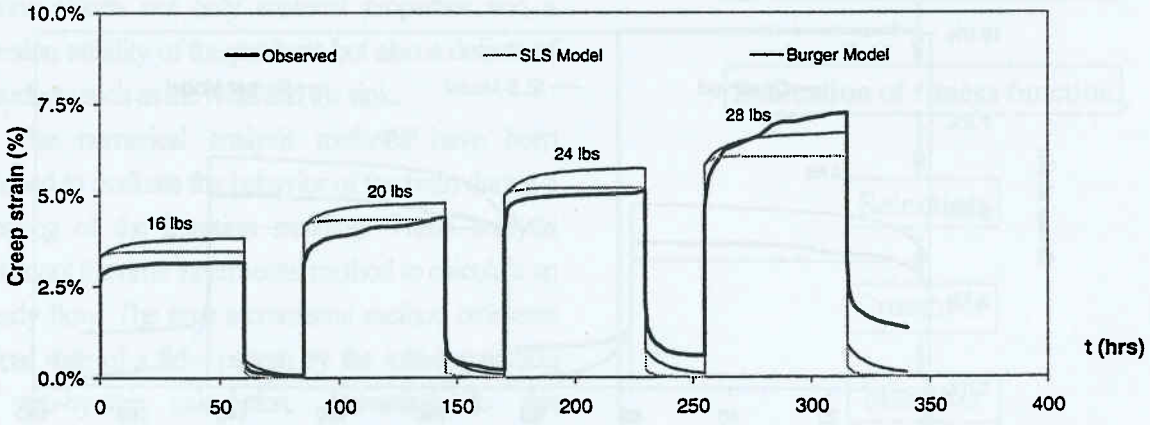

Figure 6. Observed and predicted creep and recovery behaviour of a parallel nylon 66 & HDPE bipolymer

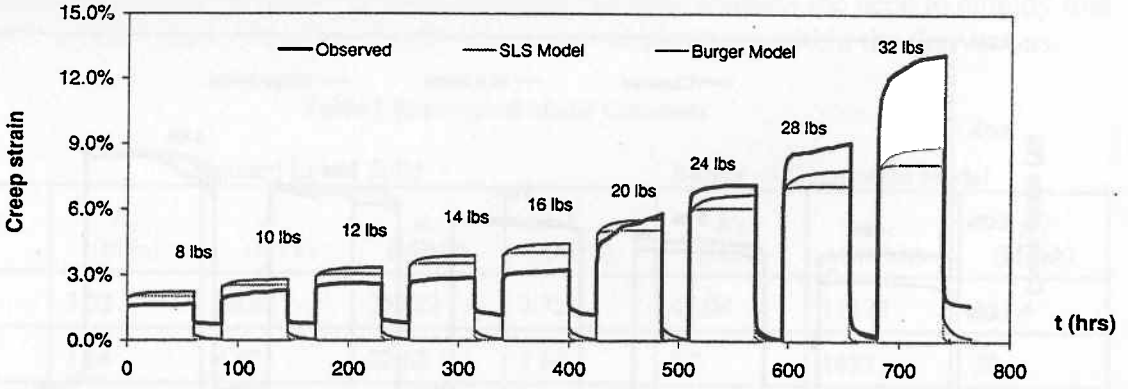

Figure 7. Observed and predicted creep and recovery behaviour of a parallel nylon 66 & LDPE bipolymer

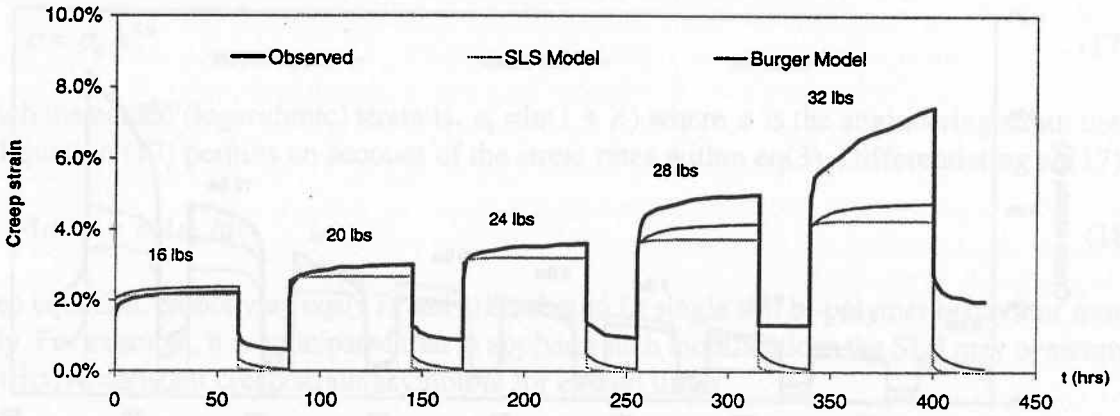

Figure 8. Observed and predicted creep and recovery behaviour of a parallel nylon 66 & polypropylene bipolymer

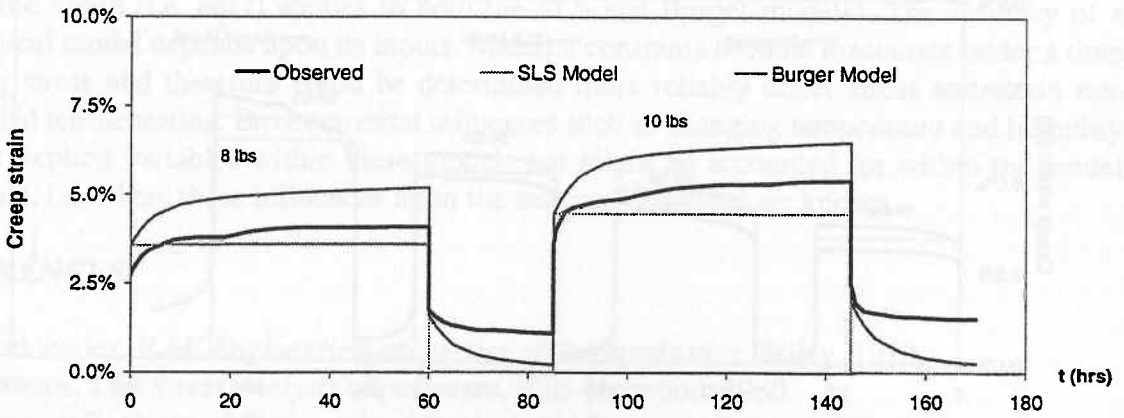

Figure 9. Observed and predicted creep and recovery behaviour of a parallel HDPE & LDPE bipolymer