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INTERFACE BEHAVIOR BETWEEN A GRANULAR BODY AND A ROUGH WALL UNDER SHEARING

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SUMMARY: This paper presents the results of numerical studies of the interface behavior between a granular body in contact with a rough wall under plane shearing. Herein the essential properties of a dry granular material are described using a hypoplastic Cosserat continuum approach. Along the bounding wall in motion the slide and rotation resistance of particles in contact is mainly determined by the interaction between the wall roughness and the size, shape and roughness of the grains. It is demonstrated in the paper that with the so-called Cosserat rotations the rotation resistance between the granular layer and the surfaces adjoining the boundaries of the granular body can be modeled in a physically natural manner. The numerical investigations show that for large shearing the deformation is localized within a narrow zone parallel to the interface. As a result of strain localization an initially isotropic material gets a transversely isotropic structure.

KEYWORDS: Cosserat continuum, hypoplasticity, interface behavior, granular materials, plane shearing, shear localization.

INTRODUCTION

The localization of shear deformations within a narrow band parallel to a bounding structure in motion is a well-known phenomenon in granular materials [1,2]. Herein the evolution of the localized zone mainly depends upon the grain size distribution and grain shape, the stress level and initial density, the slide and rotation resistance of particles in contact with the bounding structure and the dilatancy resistance of the whole system. The displacement profile across the height of the shear band is no longer linear as predicted by a classical continuum approach. In the present paper shearing of an granular strip with respect to the interface behaviour of the bounding structure is modeled based on a hypoplastic Cosserat continuum [3,4,5]. The evolution equations for the stress tensor and the couple stress tensor are incrementally non-linear, which models unelastic behavior. The constitutive equations take into account the influence of the pressure level, the void ratio, the mean grain diameter and the rotation resistance of grains. With the Cosserat boundary conditions the rotation resistance of particles at the interface between the granular layer and the surfaces adjoining the boundaries of the granular body can be modeled in a natural manner. Very rough walls can capture the adjoining small grains so that neither sliding nor rotating may occur. Then the relative displacement and the Cosserat rotation at the interface are zero. For medium rough boundaries and quasi-static processes an empirical relation between the boundary displacement and the corresponding Cosserat rotation is assumed [4].

HYPOPLASTIC COSSERAT MODEL

In a Cosserat continuum a material point possesses displacement degrees of freedom u_i (i=1,2,3) and rotational degrees of freedom ω_i^c (i=1,2,3) which are called Cosserat rotations. The rate of deformation and the rate of curvatures are defined as $\dot{\varepsilon}_{ij} = \partial \dot{u}_i / \partial x_j + \epsilon_{kij}\dot{\omega}_k^c$ and $\dot{\kappa}_{ij} = \partial \dot{\omega}_i^c / \partial x_j$ respectively, where $\partial \dot{u}_i / \partial x_j$ denotes the velocity gradient and ϵ_{ijk} denotes the permutation tensor. The proposed hypoplastic Cosserat model includes three state variables, i.e. the stress tensor σ, the couple stress tensor μ and the void ratio e. The evolution of the state variables are described by the following objective rate type equations [3,4]:

$$\mathring{\sigma}_{ij} = f_s \left[\hat{a}^2 \dot{\varepsilon}_{ij} + (\hat{\sigma}_{kl}\,\dot{\varepsilon}_{kl} + \hat{\mu}_{kl}\,\dot{\bar{\kappa}}_{kl})\,\hat{\sigma}_{ij} + f_d\,\hat{a}\,(\hat{\sigma}_{ij} + \hat{\sigma}_{ij}^*)\sqrt{\dot{\varepsilon}_{kl}\,\dot{\varepsilon}_{kl} + \dot{\bar{\kappa}}_{kl}\,\dot{\bar{\kappa}}_{kl}} \right], \tag{1}$$

$$\mathring{\mu}_{ij} = f_s\,d_{50} \left[a_c^2\,\dot{\bar{\kappa}}_{ij} + \hat{\mu}_{ij}\,(\hat{\sigma}_{kl}\,\dot{\varepsilon}_{kl} + \hat{\mu}_{kl}\,\dot{\bar{\kappa}}_{kl} + 2\,f_d\,a_c\sqrt{\dot{\varepsilon}_{kl}\,\dot{\varepsilon}_{kl} + \dot{\bar{\kappa}}_{kl}\,\dot{\bar{\kappa}}_{kl}}) \right], \tag{2}$$

$$\dot{e} = (1+e)\,\dot{\varepsilon}_{kk}, \tag{3}$$

with the normalized quantities $\hat{\sigma}_{ij} = \sigma_{ij}/\sigma_{kk}$, $\hat{\sigma}_{ij}^* = \hat{\sigma}_{ij} - \delta_{ij}/3$, $\hat{\mu}_{ij} = \mu_{ij}/(d_{50}\,\sigma_{kk})$ and $\dot{\bar{\kappa}}_{ij} = d_{50}\,\dot{\kappa}_{ij}$. Herein δ_{ij} is the Kronecker delta and d_{50} denotes the mean grain diameter, which enters the constitutive model as an internal length. The influence of the mean pressure σ_{kk} and the void ratio on the incremental stiffness and on the dilatancy behavior is modeled by the factors f_s and f_d respectively. These factors are functions of the current void ratio e and the pressure dependent maximum void ratio e_i, minimum void ratio e_d and critical void ratio e_c, i.e.

$$f_s = \left(\frac{e_i}{e}\right)^{\beta} f_b \tag{4}$$

and

$$f_d = \left(\frac{e - e_d}{e_c - e_d}\right)^{\alpha}, \tag{5}$$

with

$$\frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \frac{e_i}{e_{i0}} = \exp\left[-\left(\frac{\sigma_{kk}}{h_s}\right)^{n}\right]. \tag{6}$$

Herein n, h_s, α, β, e_{i0}, e_{d0} and e_{c0} are constitutive constants and f_b can be derived from a consistency condition [4]. Factor $\hat{a}$ in Eqn. 1 and factor a_c in Eqn. 2 are related to stationary states which can be reached asymptotically under large shearing. $\hat{a}$ depends on the so-called angle of internal friction φ_c, and on the symmetric part of the normalised stress deviator, i.e. $\hat{\sigma}^{*s}_{kl} = (\hat{\sigma}^*_{kl} + \hat{\sigma}^*_{lk})/2$. The proposed function for $\hat{a}$ reads [5]:

$$\hat{a} = \frac{\sin\varphi_c}{3 - \sin\varphi_c}\left[\sqrt{\frac{8/3 - 3\,(\hat{\sigma}^{*s}_{kl}\,\hat{\sigma}^{*s}_{kl} + \hat{\sigma}^{*s}_{kl}\,\hat{\sigma}^{*s}_{lm}\,\hat{\sigma}^{*s}_{mk})}{1 - 3\,\hat{\sigma}^{*s}_{kl}\,\hat{\sigma}^{*s}_{lm}\,\hat{\sigma}^{*s}_{mk}/(\hat{\sigma}^{*s}_{kl}\,\hat{\sigma}^{*s}_{kl})}} - \sqrt{\hat{\sigma}^{*s}_{kl}\hat{\sigma}^{*s}_{kl}}\right]\,. \tag{7}$$

Altogether the constitutive model includes 11 constants which are closely related to granular properties, i.e. they can be estimated from the grain size distribution, the grain shape and the grain hardness [6]. For the present numerical investigations the following material constants are used: $e_{i0} = 1.2$, $e_{d0} = 0.51$, $e_{c0} = 0.82$, $\varphi_c = 30°$, $h_s = 190$ MPa, $\alpha = 0.11$, $\beta = 1.05$, $n = 0.4$, $d_{50} = 0.5$ mm, $a_c = 1.0$.

MODELING THE INTERFACE BEHAVIOR

Apart from stress and displacement boundary conditions of a non-polar continuum additional non-standard boundary conditions, i.e. couple stresses and Cosserat rotation boundary conditions, must also be defined for the present model. For rough and medium rough boundaries and pure translatoric motion the following assumptions are made to model the interface behaviour in a simplified manner:

i.) Boundary particles of the granular body are permanently in contact with the adjoining structure, thus the relative displacement of boundary particles perpendicular to the adjoining structure surface is zero.

ii.) The component u_p of the particle diplacement parallel to the surface of the adjoining structure is equal to or less than the parallel displacement of the adjoining structure boundary u_b, i.e.

$$u_p = f_u\,u_b\,. \tag{8}$$

Herein the dimensionless factor $0 \leq f_u \leq 1$ denotes the fraction of u_b which is transmitted to the boundary of the granular body. With respect to (8) the relative displacement u_r then reads: $u_r = u_b - u_p = (1 - f_u)\,u_b$, which can also be represented as the sum of the part $u_{rr} = f_r\,u_b$ due to grain rotation and the part $u_{rs} = f_s\,u_b$ due to sliding, i.e.

$$u_r = u_{rr} + u_{rs} = (f_r + f_s)\,u_b\,. \tag{9}$$

f_r and f_s in Eqn. 9 denote the fractions of u_b which are transmitted as rotation and sliding respectively. Herein the condition $f_u + f_r + f_s = 1$ holds for consistency.

iii.) Between the Cosserat rotation ω^c_p at the boundary of the granular body, the mean grain diameter d_{50}, the factor f_r and the displacement u_b the following relation was proposed [3]:

$$\omega^c_p = f_r\,\frac{u_b}{d_{50}/2}\,. \tag{10}$$

With respect to Eqn. 8 relation 10 for ω^c_p can alternatively be represented as a function of the boundary displacement u_p [4], i.e.

$$\omega^c_p = \frac{f_r}{f_u}\,\frac{u_p}{d_{50}/2}\,. \tag{11}$$

NUMERICAL SIMULATIONS

In the following the influence of the Cosserat boundary conditions at the top surface of an infinite granular strip under plane shearing is investigated. For the numerical simulation the present hypoplastic Cosserat model was implemented in a finite element code using a four-node element with bilinear shape functions to describe the displacements and Cosserat rotations within the element [5]. In the case of plane strain conditions only three degrees of freedom remain for each node, i.e. u_1, u_2 and ω_3^c, as shown in Fig. 1.

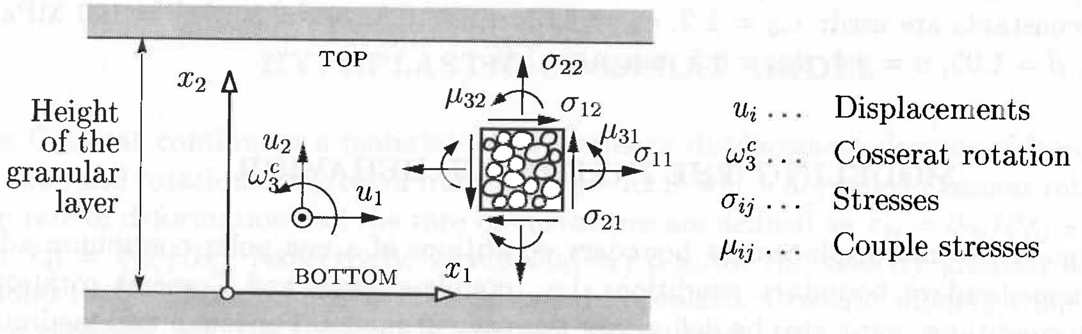

Figure 1: Modeling a plane layer of granular material with a Cosserat continuum.

All calculations are performed for a shear layer with a height of 4 cm and starting from same homogeneous and isotropic initial states, i.e. $e_o = 0.6$, $\sigma_0 = -100$ kPa and $\mu_0 = 0$. The bottom of the granular layer is fixed, i.e. $u_{1_B} = u_{2_B} = 0$ and $\omega_{3_B}^c = 0$, and at the top surface a vertical pressure of $\sigma_{22} = -100$ kPa is kept constant. A shear deformation is initiated by prescribed horizontal node displacements u_{1_T} while the vertical displacement is obtained as a result of the dilatancy behavior within the whole specimen.

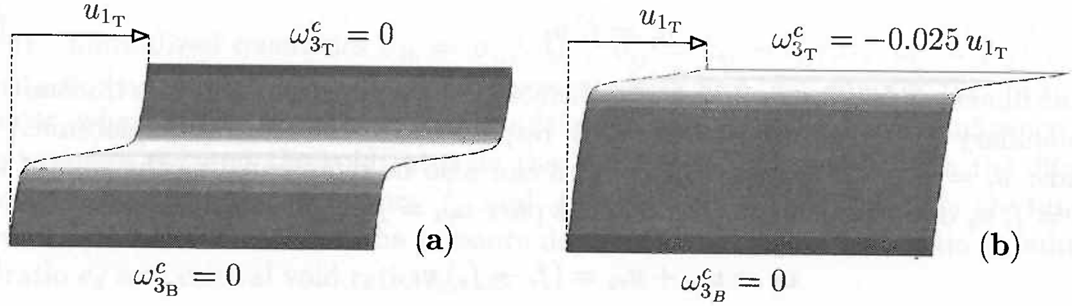

Figure 2: Influence of the Cooserat boundary conditions on the distribution of the deformation and of the void ratio.

Very rough walls can capture the adjoining grains so that neither sliding nor rotating may occur. This case can be modeled by locked Cosserat rotations at the top surface, i.e. $\omega_{3_T}^c = 0$. The simulation shows that the deformation is localized within a narrow zone in the middle of the layer (Fig. 2a). The light strip indicates a higher void ratio as a result of dilatancy in the localized zone. For shear displacements of 0.4, 1, 2 and 3 cm at the top of the layer the distribution of the horizontal displacements u_1, the void ratio e and the Cosserat rotations ω_3^c across the height of the layer are shown in Fig. (4a, 4b, 4c). It can clearly be seen that the void ratio e and the Cosserat rotation ω_3^c increases within the localized zone while outside this zone these quantities remain almost unchanged. The distribution of the stress components σ_{11} and σ_{21} is influenced by the polar effect while

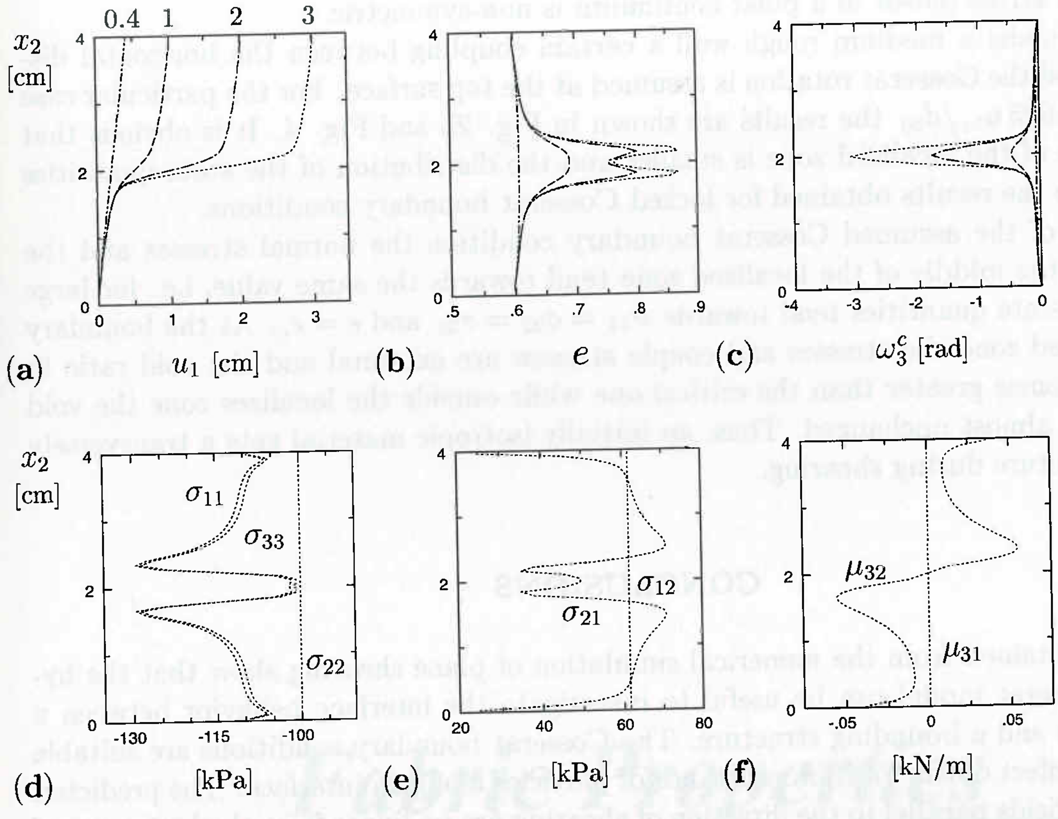

Figure 3: Plane shearing between two very rough walls

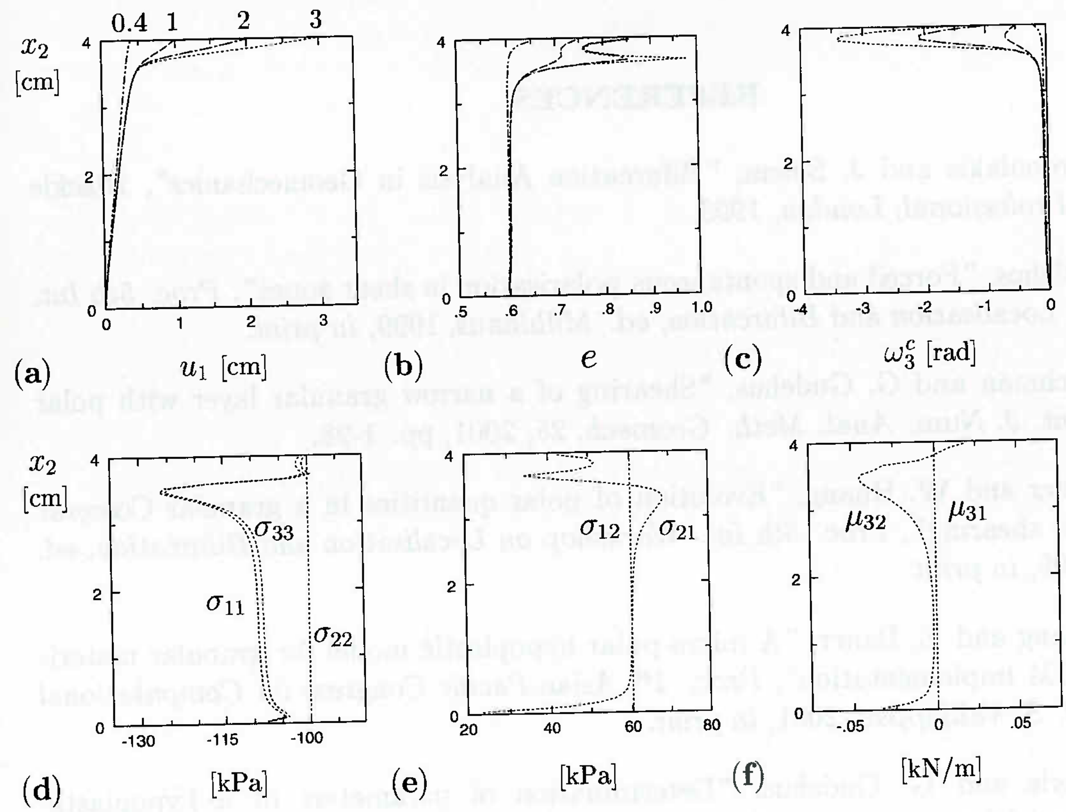

Figure 4: Plane shearing between a very rough bottom and a medium rough top wall

σ_{22} and σ_{12} are constant as required for the equilibrium (Fig. 4d and Fig. 4e). Since $\sigma_{12} \neq \sigma_{21}$ the stress tensor in a polar continuum is non-symmetric.
In order to model a medium rough wall a certain coupling between the horizontal displacement and the Cosserat rotation is assumed at the top surface. For the particular case of $\omega^c_{3_T} = -0.025\, u_{1_T}/d_{50}$ the results are shown in Fig. 2b and Fig. 4. It is obvious that the thickness of the localized zone is smaller and the distribution of the state quantities is different to the results obtained for locked Cosserat boundary conditions.
Independent of the assumed Cosserat boundary condition the normal stresses and the void ratio in the middle of the localized zone tend towards the same value, i.e. for large shearing the state quantities tend towards $\sigma_{11} = \sigma_{22} = \sigma_{33}$ and $e = e_c$. At the boundary of the localized zone the stresses and couple stresses are extremal and the void ratio in this zone becomes greater than the critical one while outside the localizes zone the void ratio remains almost unchanged. Thus, an initially isotropic material gets a transversely isotropic structure during shearing.

CONCLUSIONS

The results obtained from the numerical simulation of plane shearing show that the hypoplastic Cosserat model can be useful to investigate the interface behavior between a granular body and a bounding structure. The Cosserat boundary conditions are suitable to model the effect of the rotation resistance of particles along an interface. The predicted displacement fields parallel to the direction of shearing are no-linear from the beginning of shearing. For large shearing the deformation is localized within a narrow zone. For very rough boundaries the localized zone occurs in the middle of the shear layer, otherwise it is located near the smoother boundary.

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