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SHEAR BUCKLING OF COMPOSITE DRIVE SHAFTS UNDER TORSION

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SUMMARY: In this research trosional stability of composite drive shafts under torsion is studied. Composite materials are considered as the suitable choice for manufacturing of long drive shafts. The applications of this kind of drive shafts are developed in various products such as, cars, helicopters, cooling towers, etc.

From the design point of view, local and global trosional instability of drive shafts limits the capability of torque transferring of them. In this study, after reviewing the closed form solution methods to calculate the buckling torque of composite drive shafts, a finite element analysis is performed to study the behavior of them. Furthermore, to evaluate the results obtained by finite element method a comparison with experimental and analytical results is presented. A case study of the effects of boundary conditions, fiber orientation and stacking sequence on the mechanical behavior of composite drive shafts is also performed.

Finally, the reduction of the trosional natural frequency of a composite drive shaft due to increase of applied torque is studied.

KEYWORDS: Composite Drive Shafts, Torsion, Finite Element, Shear Buckling, Natural Frequency.

INTRODUCTION

The general stability of drive shafts under torsion is studied by many researchers. Greenhill [1] for the first time in 1883 presented a solution for torsional stability of long solid shafts. This method of solution can be used for calculating of the first torsional buckling mode of the hollow shafts and tube. The first and oldest buckling analysis of thin-walled cylinders under torsion is presented by Schwerin [2] in 1924. But his analysis did not show a good agreement with experimental results.

In 1931 Kubo and Sezawa [3] presented a theory for calculating the torsional buckling of tubes and also reported on experimental results for rubber models. However, this theory did not show an agreement with experimental results. Lundquist [4] preformed an extensive experiment on strength of Aluminum shafts under torsion. The results of the experiments were reported in 1932.

While there was no analytical solution till 1933 for simulation of the buckling behavior of drive shafts, the experimental results were the only basis for the research of Donell [5]. In 1934 he presented a theoretical solution for instability of drive shafts under torsion. He used the theory of thin-wall shells for analysis. He evaluated his theory with available experimental results, including about fifty tests. His studies show that the torsional failure load measured by experiments is always less than that obtained by theory. The main reason for that is the initial eccentricity of the shafts in the experiments. All of the mentioned studies were limited to isotropic materials. Other researchers such Timoshenko, Flugge, and Batdrof performed extensive studies on this field for isotropic materials.

General theory of isotropic shells is presented for the first time by Ambartsumyan [6] and Fong [7] in 1964. Ho and Cheng [8] performed a general analysis on the buckling of non-homogeneous anisotropic thin–wall cylinder under combined axial, radial and torsional loads by considering four boundary conditions. Chehil and Cheng [9] studied the elastic buckling of composite thin-wall shell cylinders under torsion based on the large deflection theory of shells.

Tennyson [10] using a theoretical method studied the classical linear elastic buckling of nonisotropic composite cylinders "perfect" and "imperfect" under different loading conditions. He compared his results with experiments. Bauchau *et. al.* [11] in 1988 measured the torsional buckling load of some composite drive shafts made of Carbon/Epoxy they predicted the torsional buckling load using shell theory by considering the effects of elastic coupling and transverse shear deformation very well.

Bert and Kim [12] in 1995 performed a theoretical analysis on torsional buckling of the composite drive shafts. They predicated the torsional buckling load of composite drive shafts with various lay-ups with good accuracy by considering the effect of off-axis stiffness and flexural moment. This theory can predict the torsional buckling of composite drive shaft under pure torsional and combined torsion and bending.

Chen and Peng [13] in 1998 using a finite element method studied the stability of composite shafts under rotation and axial comparison load. They predicted the critical axial load of a thin-wall composite shaft under rotation.

PROBLEM STATEMENT

When a hollow shaft is subjected to torsion, for a certain amount of torsional load instability occurs. It is called torsional buckling load. Therefore, considering the torsional buckling load is important in design of drive shafts. This parameter is more critical in design of composite shafts. Because the composite drive shafts are made longer. Although increasing the length of drive shaft does not change the static torsional strength, it can decrease the torsional buckling load capacity of the shafts. Therefore, the calculation of torsional buckling load for composite drive shafts is very important. In the following section it is shown that torsional buckling strength of a shaft must be higher than the static torsional strength.

Secondly, the stacking sequence of the layers effects the torsional buckling capacity of drive shafts. Therefore, selection a suitable stacking sequence can increase the torsional buckling of the composite shafts.

Thirdly, in general the composite drive shafts have a lower torsional buckling capacity in comparison with metallic shafts for the same geometry. An important reason for that is the existence of interlaminar shear stresses and the coupling between the in-plane and out-of-plane stresses for composite shafts. In a metallic shaft under torsion, the shear stress is the only existing stress, however, for a composite shaft all stresses can exist.

ANALYTICAL RELATIONS TO CALCULATE THE TORSIONALBUCKLING OF COMPOSITE SHAFTS

In design of a composite shaft, before applying a finite element technique, a closed form solution is needed. In order to have a first guess for a design, a simple equation is needed to calculate the torsional buckling load of a long thin-wall shaft. There are various equations for this purpose in the literature. These equations are empirical obtained based on experimental studies. In the following two equations used by many others are presented. The first equation is presented in reference [14].

$$T_{buckling} = \frac{2.289}{\sqrt{L}} \times E_1^{0.375} \times E_2^{0.625} \times t^{2.25} \times D^{1.25} \tag{1}$$

the second equation is presented in reference [15].

$$T_{buckling} = \frac{1.854}{\sqrt{L}} \times E_1^{0.375} \times E_2^{0.625} \times t^{2.25} \times D^{1.25} \tag{2}$$

in these equation t is the thickness, D is the average diameter, L is the length of the shaft and E_1 and E_2 are the longitudinal and transverse stiffness of the shaft, respectively.

Based on these equations, the torsional buckling load is maximized for a 45° orientation of fiber, while E_1 and E_2 are equal. Suppose the stiffness of composite material is equal to that of the steel, i.e., 200 GPa. So, for a 45° orientation of fibers, the longitudinal and transverse stiffness (E_1 and E_2) of the composite shaft is much less than 200 GPa. This shows if a metallic shaft is replaced by a composite shaft, for the same geometry, the thickness of the composite shaft is bigger. To have a similar torsional buckling load for both shafts, the thickness of composite shaft must be increased. By considering the low density of composites this does not increase the weight of the composite shafts too much.

FINITE ELEMENT ANALYSIS TO CALCULATE THE TORSIONAL BUCKLING OF COMPOSITE SHAFTS

In this research finite element analysis is performed using ANSYS software. To model the composite shaft, shell 99 element is used and the shaft is subjected to torsion. The shaft is fixed in one end in axial, radial and tangential directions and is subjected to torsion on the other end. After static analysis of the shaft, the stresses are saved in a file to calculate the buckling. The output of the buckling analysis is a load coefficient which is the ratio of buckling load to the static load. This software calculates the modes of buckling of the composite shaft. In Figs.1 and 2, the mesh configuration and the first and the second modes of buckling of the composite shaft are shown.

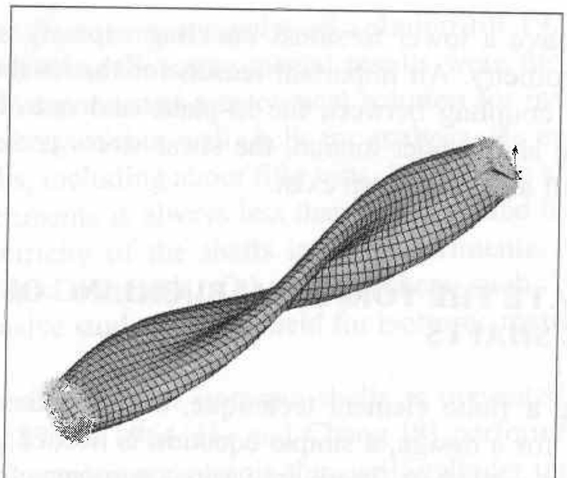

Figure 1 First Mode of Buckling of the Composite Shaft

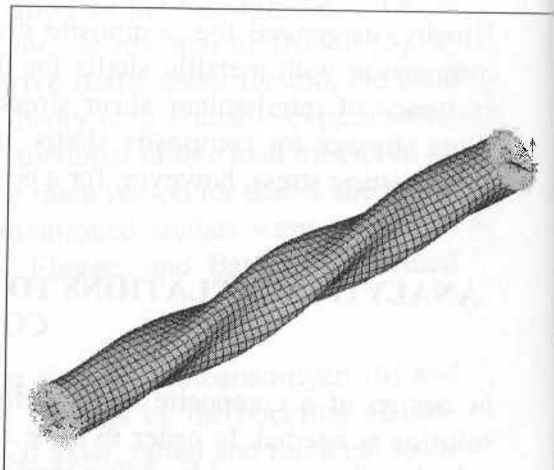

Figure 2 Second Mode of Buckling of Composite Shaft

In Table 1 the results of the buckling torque obtained from closed form solution, finite element analysis and experimental results are compared. The results are presented for Carbon/Epoxy composite shaft with different ply sequences and the material properties shown in Table 2. In Table 1, two important points are shown. First, the ply sequence has important effect on the torsional buckling of the shaft. Second, the results obtained from finite element analysis in this research show good agreement with experimental results. In most cases the results obtained by this research show a better agreement with experimental results compared with the methods presented in [11,12].

Table 1 Buckling Analysis of Composite Shaft

Shaft no.	Lay-up definition	L (m)	R (mm)	Buckling torque (N.m)			
				*a**	*b**	*c**	*d**
1	15, -15, -45, -15, 15, +45	0.26	41.27	486	523	535	472
2	-45, -15, 15, 45, 15, -15	0.26	41.27	350	366	382	372
3	30, -30, 30, -30, 30, -30	0.32	37.19	390	535	394	395
4	45, -45, 45, -45, 45, -45	0.32	37.42	490	490	540	460
5	0, 0, 45, -45, 45, -45, 0, 0	0.32	37.42	543	540	671	670

a : Experimental result from Ref. [11]*
b : Theoretical prediction for simply supported edges from Ref. [11]*
c : Sanders thin shell theory from Ref. [12]*
d : Present prediction using ANSYS software*

Table 2 Mechanical Properties of Carbon/Epoxy

$E_{xx} = 134\ GPa$	$G_{xy} = 4.6\ GPa$
$E_{yy} = 8.5\ GPa$	$\nu_{xy} = 0.29$
Layers Thickness = 0.1334 mm	

EFFECTIVE PARAMETER ON TORSIONAL BUCKLING OF COMPOSITE SHAFTS

In design of composite shafts the effect of fiber orientation on torsional buckling load must be considered. In Table 3, the effect of fiber orientation on the torsional buckling load of a composite shaft is presented. In this table the results presented by reference [12] and the finite element analysis performed in this research are compared. The mechanical properties of the composite materials are show in Table 4.

Table 3 Variation of Torsional Buckling Load with Fiber Orientation

Ply orientation angle (degree)		0	15	30	45	60	75	90
Buckling torque (N.m)	Ref. [12]	1587	974	1126	1790	2617	3156	3016
	Present research	2100	1984	1320	1550	2140	2793	2950

Table 4 Mechanical Properties of the Composite Shaft

Thickness of layer	0.132 mm	$E_{xx} = 211\,GPa$
Number of layers	10	$E_{yy} = 24.1\,GPa$
Length	2.47 m	$G_{xy} = 6.89\,GPa$
Average diameter	12.57 cm	$V_{xy} = 0.36$

Although in some cases there are some differences between the results obtained in this research and results from reference [12], however, as shown in Table 3, the results obtained from two methods are in good agreement.. By comparing these results with experimental results in Table 3, it is clear that the result obtained in this research show better agreement with experimental results. The boundary condition of two ends of the shaft has a minor effect on the torsional buckling load [12]. To clarify this postulation, a Boron/Epoxy composite drive shaft with four different boundary conditions is analyzed [12] and the results presented on Table 5.

Table 5 Torsional Buckling Load of a Boron/Epoxy Composite Drive Shaft with Four Boundary Conditions using Sander's Shell Theory

Boundary condition	Buckling torque (N.m)
1) Simply supported at both ends without Axial constraint (Freely supported)	3481
2) Simply supported at both ends with Axial constant.	3664
3) Clamped at both ends	3665
4) Clamped end simply supported	3561

COMPARISON BETWEEN FINITE ELEMENT AND ANALYTICAL METHODS

To evaluate the accuracy of Eqs. (1) and (2), torsional buckling load of a shaft is calculated using these equations. The results are compared with result obtained by finite element methods. The mechanical properties of the material and the geometry of the shaft are summarized on Table 6.

Table 6 Mechanical Properties and Geometry of a Composite Shaft

$E_{xx} = 102\ GPa$	t = 3 m	$r_m = 0.035\ m$
$E_{yy} = 8.56\ GPa$	$L = 1.56\ m$	D= 0.07 m

$$T_{buckling} = \frac{2.289}{\sqrt{1.56}} \times (102\times10^9)^{0.375} \times (8.56\times10^9)^{.625} \times (0.003)^{2.25} \times (0.07)^{1.25} \qquad (1)$$

$$T_{buckling} = 3000 N.m$$

$$T_{buckling} = \frac{1.854}{\sqrt{1.56}} \times (102\times10^9)^{0.375} \times (8.56\times10^9)^{.625} \times (0.003)^{2.25} \times (0.07)^{1.25} \qquad (2)$$

$$T_{buckling} = 2440 N.m$$

Table 7 Comparison of Analytical and Finite Element Methods

Buckling Torque (N.m)		
Eq.(1)	Eq.(2)	FEM
3000	2440	2400

As shown in Table 7, Equation 2 shows more accuracy in predicting the torsional buckling load of a composite shaft. Therefore, this equation is used for calculation in this research.

VARIATION OF TORSIONAL NATURAL FREQUENCY OF A SHAFT DUE TO APPLIED TORQUE

The buckling of a shaft under torsion is similar to buckling of a shaft under axial load in a mathematical point of view. In a composite shaft, the torsion load creates shear and compression stresses in the layers in on-axis direction.

Another definition of axial buckling force of a shaft is the load on which the first natural frequency of the shaft becomes zero. In other words, when a beam is subjected to axial load, the first natural frequency of the first bending mode is decreased by increasing the load. In a certain amount of axial load, the magnitude of first natural frequency reaches to zero. In fact, for this reason the first natural torsional frequency of a shaft must be higher than the first natural bending frequency of that. In this way, when the shaft is under the applied torque, the natural torsional frequency of the shaft does not become less than the natural bending frequency. Normally, the critical speed of the shaft is selected based on the first natural bending frequency.

The squared natural frequency of a structure is a linear function of the applied load. It means, by increasing the axial load, the squared natural frequency decreased linearly [16]. There is a similar behavior for a shaft under torsion. In the following, using some assumptions, this subject is proved.

A structure is considered with a stiffness of [k], geometrical stiffness of [k_g], and the mass matrix of [M]. It must be mentioned that the geometrical stiffness is calculated for the unit load, and changes with the load linearly. To calculate the natural frequency of a structure under load, following eigenvalue problem should be solved:

$$\left[[k] + \lambda \left[k_g \right] - \omega^2 [M] \right] \{a\} = 0 \tag{3}$$

In which, λ is a known parameter and showing the magnitude of loading and ω is unknown. Equation 3 can be written as follows:

$$\left[[k] + \alpha\lambda_{cr} \left[k_g \right] - \omega^2 [M] \right] \{a\} = 0 \tag{4}$$

and $0 \le \alpha \le 1$. Equation 4 is written as follows:

$$\left[(1-\alpha)[k] + \alpha \left([k] + \lambda_{cr} \left[k_g \right] \right) - \omega^2 [M] \right] \{a\} = 0 \tag{5}$$

By assuming that the mode shapes of bending and torsion are the same, the second term of Eq. 5 is vanished and we have:

$$\left[(1-\alpha)[k] - \omega^2 [M] \right] \{a\} = 0 \tag{6}$$

$$\left[[k] - \frac{\omega^2}{(1-\alpha)} [M] \right] \{a\} = 0 \tag{7}$$

Equations 6 and 7 show that for a structure, with similar mode shapes for bending and torsion, the squared of natural frequency is a linear function of the load and under critical load, one of the natural frequencies becomes zero.

In Fig. 3, the variation of the squared natural frequency of a beam as a function of the load is shown. As shown in this figure, the squared natural frequency does not depend on the boundary conditions and changes linearly by increasing the load and becomes zero as load reaches to critical load. It must be mentioned that the higher natural frequencies are also decreased linearly by the axial load. However, just the first natural frequency reaches to zero at critical load and the others decreased. All the points mentioned here for a beam under bending are valid for a shaft under torsion.

To clarify the above mentioned points, the variation of the first five natural frequencies of a composite shaft in terms of the variation of the applied torque is shown in Figs. 4 and 5. These curves are drawn based on a finite element analysis. The mechanical properties and the geometry of the model are summarized in Table 8 and 9. In Figs. 4 and 5, f1 to f5 express the magnitudes of first natural frequencies of the shaft.

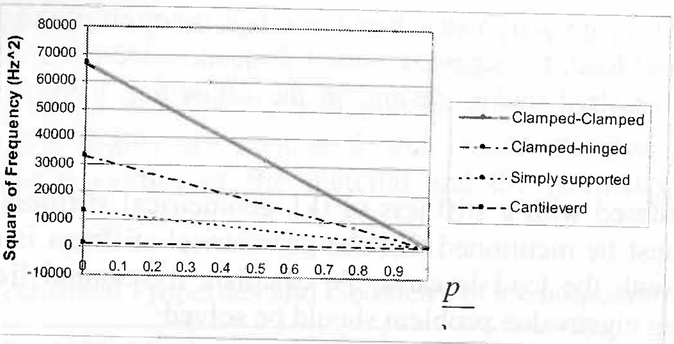

Figure 3 Variation of Squared Natural Frequency of a Beam with Different Boundary Conditions under Axial Compressive Load [16]

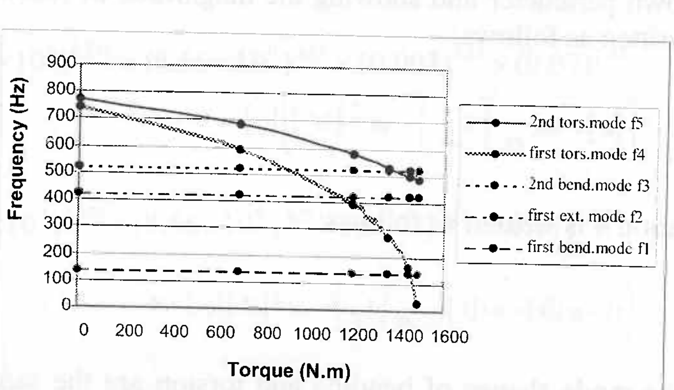

Figure 4 Variation of the first five natural frequencies of a composite shaft in terms of the applied torque

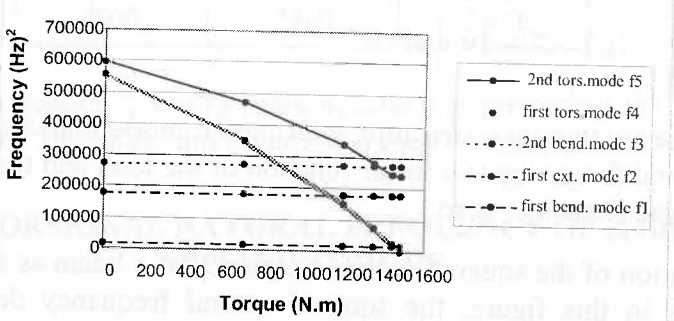

Figure 5 Variation of the squared first five natural frequencies of a composite shaft in terms of the applied torque

Table 8 Geometry and Fiber Volume Fraction of the Composite Shaft

Shaft Geometry	**Fiber Volume Fraction**	**Fiber Orientation**
L= 1.337 m t = 2.5 mm D= 63.7 mm	30% E-glass / Epoxy 70% Hs-carbon / Epoxy	11 11

Table 9: Material Properties of the Composite Shaft

Material	E_{xx} (GPa)	E_{yy} (GPa)	G_{xy} (GPa)	ν	ρ (kg/m^3)
E-Glass / Epoxy	50	12	5.6	0.3	2000
Hs-Carbon/ Epoxy	134	7	5.8	0.3	1600

As shown in Fig. 4 if we apply a torque equal to 890 N.m the fourth mode of vibration is replaced by the third mode. By increasing the applied torque, the lower modes of vibration are replaced by the higher ones. In a torque equal to 1450 N.m the first frequency of torsion (fourth mode) is equal to the first frequency of bending (first mode). Finally, in a torque equal to 1510 N.m the first natural frequency reaches to zero and the shaft is unstable at that point.

In Fig. 5, the fairly linear variation of the squared natural frequencies in terms of the applied torque is shown. This linear variation verifies the results obtained by finite element technique. The results obtained by the present research show that in design of a composite shaft, the buckling torque must be properly higher than the static applied torque.

DISCUSSION AND RESULTS

- The boundary conditions of the shaft do not effect the buckling torque too much.
- The fiber orientation of composite shaft strongly changes the buckling torque.
- The stacking sequence of the layers for a composite shaft also strongly changes the bucking torque.
- The finite element modeling presented in this analysis is able to predict the buckling torque very well.
- Composite shafts in comparison with the metallic shafts, with the same geometry, have lower bucking torque.
- By increasing the applied torque on a shaft, the squared natural frequencies of that decrease linearly.
- Increasing the applied torque decreases the natural frequencies of torsion and does not change the other modes.
- The frequency of the first mode of torsion under a certain torque which is the buckling torque becomes zero.

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