# Modeling the Viscoelastic Behavior of Fiber Reinforcing Fabrics

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**SUMMARY**: The deformation of fibrous materials plays an important role in the Liquid Composite Molding (LCM) processes. These materials display markedly non-linear viscoelastic characteristics. A model of fibrous material deformation is developed, incorporating this non-linear viscoelastic response, with a view to being used in simulations of LCM processes. The model is one-dimensional and is appropriate for the simple fiber compaction deformation which occurs during LCM processes. A limited number of compaction experiments were carried out to determine the model parameters for a continuous filament mat. These included a rapid compaction (100mm/min), a number of slower compactions to the same final volume fraction, and then a number of tests at a constant compaction speed to different final volume fractions. The model gives reasonably good results over a range of compaction speeds and volume fractions.

**KEYWORDS**: Viscoelastic, Fibers, Reinforcements, Relaxation, Modeling, Compaction, Process Modeling.

## INTRODUCTION

The Liquid Composite Molding (LCM) processes, such as Resin Transfer Molding (RTM), Injection/Compression Molding (I/CM) and Vacuum Assisted Resin Transfer Molding (VARTM) are popular processes for the manufacture of fiber-reinforced composite materials. All these LCM processes involve placement of a reinforcing fibrous material within some form of closed mold. The fibrous material is then compacted, before being impregnated with a polymer resin. The response of the fibrous material to compaction, and its subsequent response right throughout the manufacturing process, is of some importance in these processes, as it directly influences the required tooling forces, process times and other important manufacturing parameters. This paper is concerned with modeling the deformation of fibrous materials as occurs during these LCM processes.

Models of reinforcement deformation generally account for non-linear elastic deformations (e.g. [1]). The viscoelastic response of these materials has been noted many times (e.g. [2]), but the modeling of this response has been reported less often.

Earlier work has pointed to the importance of accounting for the viscoelastic response [3], particularly for processes in which the mold cavity thickness might change (e.g. in I/CM or VARTM).

The objective of this paper is to produce a simple working model of the deformation of fibrous materials, which incorporates their non-linear elastic and viscoelastic characteristics, and which can be used in simulations of the LCM processes.

## MODEL AND EXPERIMENTS

### A Model of the Compaction Tests

During preform compaction, not only are the deformations large, but the viscous effects are markedly non-linear, indicating fairly large energy changes in the material. In what follows, a model is developed which allows for this complex constitutive behavior, for the simple deformation which occurs during preform compaction. The model introduced is purely mechanical, not accounting for temperature variations; it is essentially a one-dimensional version of a finite–strain, linear viscoelastic, thermomechanical model (see, for example, [4]), only with a non-linear viscous response. First, introduce the free energy function

$$\Psi(e,\xi_i) = \Psi_\infty(e) + \sum_{i=1}^{N} \Gamma_i(e,\xi_i) \tag{1}$$

Here, $e$ is the external (observable) strain variable, and $\xi_i$ are $i = 1,\ldots,N$ internal kinematic variables, describing the viscous effects in the material. $\Psi_\infty(e)$ is the free energy at equilibrium, that is, the energy stored after all viscous effects have terminated, whereas the second term is the so-called configurational free-energy, which characterizes the non-equilibrium state. The free energy is chosen to be of the form

$$\Psi(e,\xi_i) = \Psi_\infty(e) + \sum_{i=1}^{N} \frac{E_i}{n_i+1}(e-\xi_i)^{n_i+1} \tag{3}$$

This reduces to the linear, so-called generalized Maxwell, model (a free spring in parallel with $N$ Maxwell units) when $n_i = 1$. By taking $n_i \geq 1$ one obtains a model incorporating non-linear viscous effects. Following standard thermodynamics arguments, the stress is now obtained through a differentiation:

$$\sigma = \frac{\partial\Psi(e,\xi_i)}{\partial e} = \frac{\partial\Psi_\infty(e)}{\partial e} + \sum_{i=1}^{N} E_i(e-\xi_i)^{n_i} \equiv \sigma_\infty + \sum_{i=1}^{N} q_i \tag{4}$$

The total stress is thus the equilibrium stress $\sigma_\infty$ together with $N$ viscous stresses; note that $q_i$ can be interpreted as the viscous forces/stresses, and $\xi_i$ the strain, acting in a dashpot attached to a non-linear spring.

The thermodynamic force is

$$f_{\xi_i} = -\frac{\partial \Gamma_i(e,\xi_i)}{\partial \xi_i} = E_i(e-\xi_i)^{n_i} \tag{5}$$

which can be seen to be equal to the $q_i$, so the mechanical dissipation (the rate of working of the internal stresses which lead to an energy loss) is, by definition,

$$\Phi(e,\xi,\dot{\xi}) = -\frac{\partial \Psi(e,\xi)}{\partial \xi}\dot{\xi} = \sum_{i=1}^{N} f_{\xi_i}\dot{\xi}_i = \sum_{i=1}^{n} q_i \dot{\xi}_i \tag{6}$$

Take now a viscosity law of the form

$$q_i = \eta_i (e_{\max} - e)^{-m} \dot{\xi}_i \tag{7}$$

Here, $\eta$ is the viscosity, $m$ is a material parameter, and $e_{\max}$ is the maximum theoretical strain possible in the material. It can be seen that this implies that the greater the strain (volume fraction), the more rapid the rise in stress for a given strain-rate. The idea here is that the more tightly packed are the fibers, the less room there is for fiber slippage, and hence viscous effects [2]. In the limit, of course, no slippage should be possible. Eqns. 6 and 7 imply that $\Phi = \sum \eta_i (e_{\max} - e)^{-m} \dot{\xi}_i^2$, which is positive, as required by the second law, provided $m \geq 0$.

It remains to write down the evolution (first order differential) equations for the viscous forces, which follow from $q_i = \eta_i (e_{\max} - e)^m \dot{\xi}_i = E_i (e - \xi_i)^{n_i}$:

$$\dot{q}_i = -\frac{n_i E_i^{1/n_i}}{\eta_i}\left[(e_{\max} - e)^m q_i^{2-1/n_i} - \eta_i \dot{e} q_i^{1-1/n_i}\right] \tag{8}$$

In Eqn. 8, the strain measure $e$ employed is the true (logarithmic) strain and $\dot{e}$ is the rate of deformation. In terms of volume fractions, these are ($\dot{V}_f$ is the rate of change of $V_f$)

$$e = \ln\frac{V_f}{V_{f0}}, \qquad \dot{e} = \frac{\dot{V}_f}{V_f} \tag{9}$$

**Experimental Procedure**

A study has been undertaken into the deformation characteristics of a glass-fiber continuous filament mat (CFM, 450 g/m$^2$). Each preform sample consisted of 8 layers, cut into 0.2 m squares. The samples were placed between a set of rigid, parallel plates set up in an Instron 1186 testing machine, the upper plate fixed with the cross-head moving up.

Initially, each sample was compacted with a constant force of 200 N (stress $\sigma = 5\,\mathrm{kPa}$) and allowed to compact until an equilibrium position was reached and no further deformation occurred. This low level of stress and associated viscoelastic response were deemed not to affect the viscoelastic response of the material at the much higher loads encountered during full compaction, but allowed one to define an initial volume fraction $V_{f0}$ ($\approx 0.1$), which is needed to define the practical measure of strain (Eqn. 9).

Once the initialization was complete, a constant velocity compressive strain history was applied, and the compaction force applied to the sample was recorded. This constant velocity compaction was then followed by a period in which the sample was held at constant thickness.

Samples were compacted to a volume fraction $V_f = 0.35$ at speeds of 0.035, 0.5, 2, 10 and 100 mm per minute. This final, rapid, test was conducted to determine the instantaneous response of the material, for which it was assumed that viscous effects were negligible. After compaction, the samples were held at $V_f = 0.35$ until the stress relaxation curves leveled out into a constant equilibrium stress, which occurs when all viscous effects have terminated. A further series of tests were carried out at 2 mm/min to final volume fractions of $V_f = 0.25$ and 0.45. A number of the tests were repeated to ensure the reliability of the experimental procedure.

## RESULTS AND DISCUSSION

Figures 1 and 2 show the results for a model with $N = 2$ and the data in Table 1.

| $E_1$ | $n_1$ | $E_2$ | $n_2$ | $\eta_1$ | $\eta_2$ | $m$ |
|---|---|---|---|---|---|---|
| 20 | 8 | 260 | 1 | 1e5 | 1e3 | 1 |

**Table 1 Model parameters (see Eqns. 3 and 7)**

The model gives reasonable results over a range of compaction speeds and volume fractions.

The model does not match the data perfectly. One important reason for this is that there are permanent deformations occurring in the material, which are not accounted for in the model. For example, on a micromechanical level, fibers may well nest between other fibers during loading, and might not tend to return to their original positions within the perform after removal of the load, no matter how much time has elapsed.

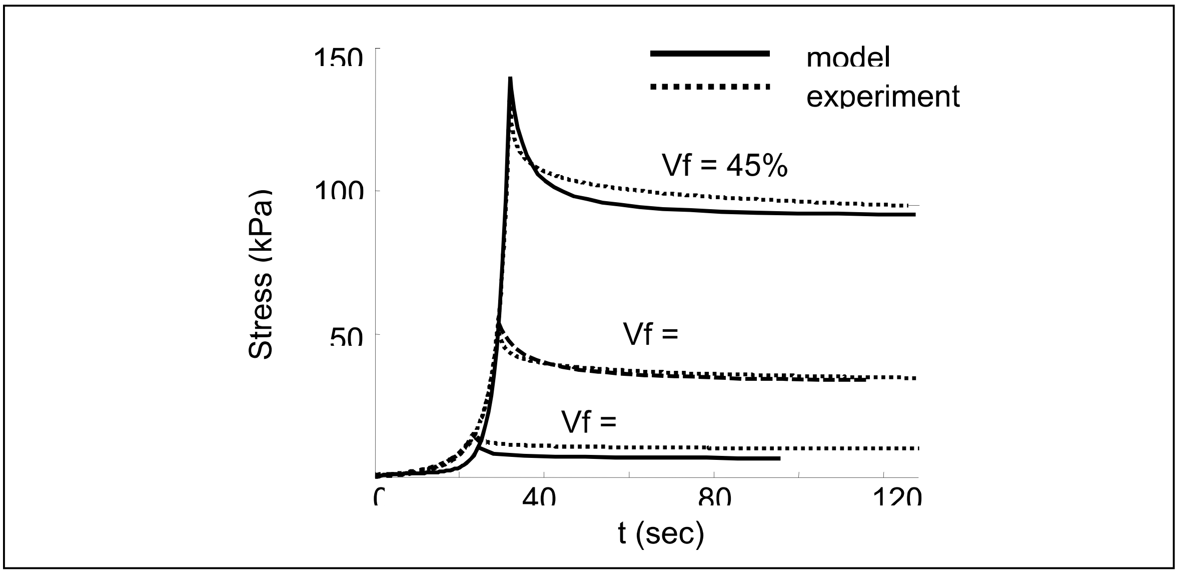


Fig. 1 Results for compaction at 2 mm/min to $V_f = 0.25$, 0.35 and 0.45

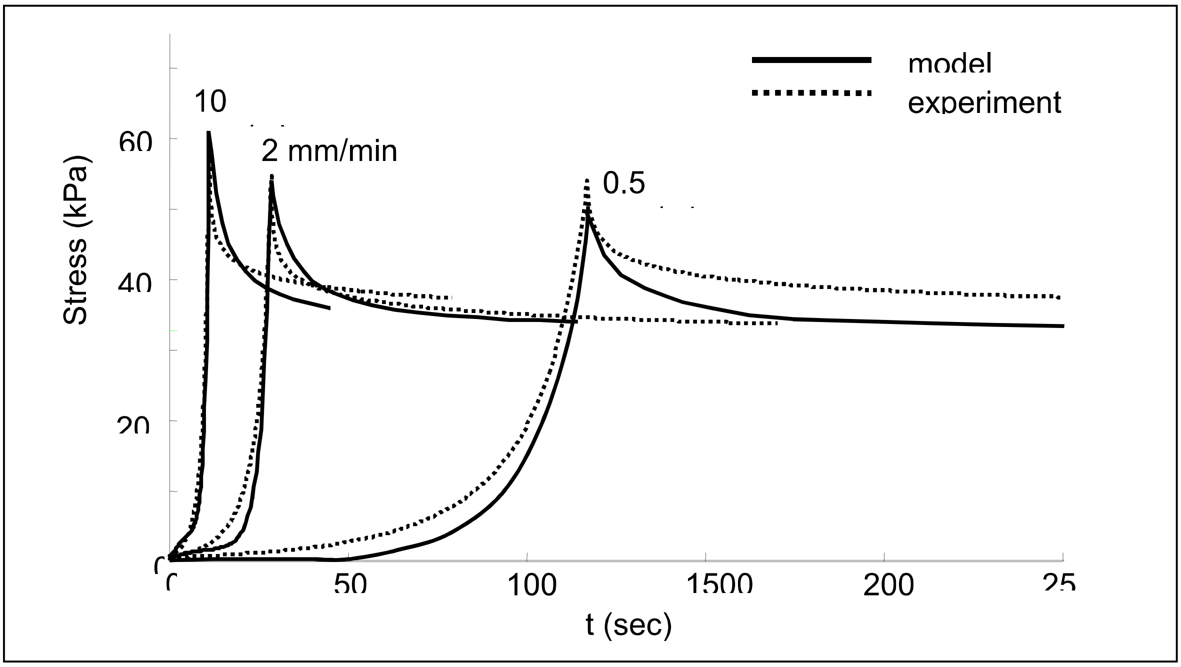


**Fig. 2 Results for compaction to $V_f = 0.35$ at different speeds**

## CONCLUSIONS

A simple one-dimensional model of fiber compaction has been developed for use in simulations of Liquid Composite Molding processes. The model requires a number of parameters, and these were obtained for a CFM mat by carrying out a limited number of tests (including a rapid compaction test, a number of compactions at different, slower, compaction speeds, and a number of compaction tests to different volume fractions). The model gives reasonable results over a range of compaction speeds and volume fractions.

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