Permeability prediction for a REV of a fibrous media with a monolithic finite element method

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ABSTRACT: Permeability is a first order parameter to model impregnation of fibre reinforcements in Liquid Composite Moulding processes, at the mesoscopic and macroscopic scales (tow or fabric scale). In this paper, we use a stabilised finite element method for numerical computation of the multiphase flows occurring at fibre (microscopic) scale. We aim to compute permeability of a representative elementary volume (REV). In our monolithic approach, interfaces between different materials are represented using level-set functions. The solid parts are taken into account by a penalisation factor assuming that the solid has a very large viscosity with respect to the fluid. The main characteristic of the monolithic approach is that we solve flow equations in the entire computational domain, including the solid part. Our results are in good agreement with analytical results found in the literature.

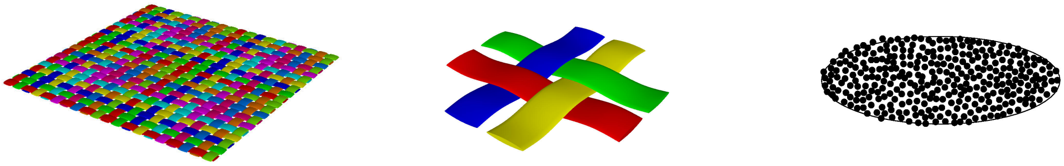
KEYWORDS: permeability, fibrous media, finite elements method

INTRODUCTION

Resin Transfer Moulding type process (RTM) is an injection process used for manufacturing large and complex composite materials with fibre reinforcement. A thermoset polymer is injected into a mould cavity which is filled with a fibrous reinforcement (mat or woven fibres, glass or carbon).

The multi-scale nature of the fibrous media (Fig. 1) makes the process difficult to model. Indeed, the reinforcement is a weaving of yarns with dimension of the order of the millimetre. A yarn can contain several thousand fibres with a diameter of the order of ten micrometres. Therefore physic taking place in this process is different for each scale.

Numerical simulation of Darcy law is used at macroscopic scale to optimize this process [1] and to give good predictions of flow front progression, filling time and injection pressure and to improve the design of tools and moulds. Numerical computations give good prediction of dry zone due to bad placement of vents.



(a) Macroscopic scale, entire part.

(b) Mesoscopic scale, the reinforcement

(c) Microscopic scale, the yarn

Fig. 1 The three scales in the RTM process.

In this study, we propose a numerical method for predicting permeability of a fibrous media. In a first part, we recall equations of the problem and present the monolithic approach, and the finite element resolution. Then, we give results of permeability computations by studying the effect of fibres volume fraction.

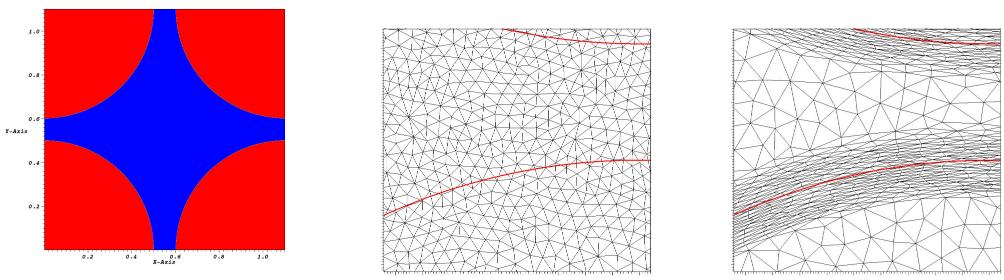
MONOLITHIC APPROACH FOR FINITE ELEMENT METHOD

Injected polymer is considered as a Newtonian incompressible fluid. Under the hypothesis of small flow-rate injection, we neglect the inertia terms, and use Stokes equations:

$$\begin{cases} \nabla p - \eta \Delta v = 0 \text{ in } \Omega \\ \nabla \cdot v = 0 \text{ in } \Omega \\ \text{B.C. on } \partial\Omega \end{cases} \tag{1}$$

where Ω is the fluid domain, $\partial\Omega$ the boundary, v the velocity, p the pressure and η the dynamic viscosity.

In the monolithic approach, a unique equation is solved on a global eulerian mesh of whole domain. All phases of the multidomain problem [2] are implicitly represented by level-set function α (Fig. 2). The interface between two phases is defined by α=0. To obtain an accurate representation of the interface, efficient anisotropic remeshing tools are used.



(a) Computed domain: four quarter fibres and a fluid zone

(b) Isotropic mesh. Interface between fluid and solid is represented by the solid line.

(c) Anisotropic mesh. Anisotropy follows the curvature of the interface.

Fig. 2 Global computational domain. Interfaces are not represented with nodes in the global mesh.

The finite element formulation is given by the weak form of Eqn. 1. The problem consists in finding (v,p) in $\left(H_0^1(\Omega)\right)^d \times L^2(\Omega)$ such as:

$$\begin{cases} \int_\Omega p\nabla \cdot w d\Omega - \int_\Omega 2\eta\varepsilon(v) : \varepsilon(w) d\Omega = 0 \quad \forall w \in \left(H_0^1(\Omega)\right)^d \\ \int_\Omega q\nabla \cdot w d\Omega = 0 \quad \forall q \in L^2(\Omega) \end{cases} \tag{2}$$

where $\varepsilon(v)=1/2(\nabla v+\nabla^T v)$. $H_0^1(\Omega)$ is a subspace of $H^1(\Omega)=\{f\in L^2(\Omega),\nabla f\in L^2(\Omega)\}$ in which the test function become zero on the boundary, and with q in $L^2(\Omega)=\{f, \int_\Omega f^2\, d\Omega < \infty\}$ and d the space dimension.

Materials properties are defined on the entire domain. We define a characteristic function I_s of the solid phase. This function has value 0 in the fluid phase and unity in the solid phase.

$$I_s(x,t) = \begin{cases} 1 \ \ if\ \alpha(x) > e \\ \frac{\alpha}{e} \ \ if\ 0 < |\alpha(x)| < e \\ 0 \ \ if\ \alpha(x) < -e \end{cases} \tag{3}$$

where e is the half thickness of the mixing zone centred around the interface. This thickness depends on the mesh size in the neighbourhood of the interface. The quality of the discretisation of this function is directly linked to the mesh refinement of the interface. We have then $\eta=\eta_s I_s+\eta_f(1-I_s)$ with η_s and η_f the solid and fluid viscosities respectively. We impose high viscosity to the solid part (i.e. $\eta_s=10^3\eta_f$). This viscosity term acts as a penalty term in Eqn. 2 in order to take into account of rigidity of the solid part [2]. Resolution is done using mini-element P1+/P1 (linear for pressure and linear + bubble for velocity).

Darcy law is a model for flow in porous media. We assume that the porous media is saturated. We write the Darcy equation by neglecting inertia and viscosity effect in front of resistance induced by the solid structure of the porous media and averaging microscopic momentum equation (Eqn. 1) [3]:

$$\langle v \rangle = -\frac{K}{\eta}\nabla\langle p\rangle^f \tag{4}$$

where K is the permeability [m²], $\langle\cdot\rangle$ the volume average and $\langle\cdot\rangle^f$ the average taken on the fluid volume. The Darcy law gives the permeability using results of microscopic simulations.

Permeability is usually obtained by experimental measurements. Predicting this parameter is a long time challenge. Several analytic relations have been proposed. Some of them are based on the lubrication approximation [4,5]. In this case, fibres are regularly packed. A periodic REV is chosen, and Stokes equations are solved, with lubrication approximation and zero velocity on fibres and periodic boundary conditions for the REV boundaries. Others [6] are based on cell model. The medium can be divided in independent cells. The REV is a cell containing one fibre. Stokes equations are solved with suitable boundary conditions on external REV boundaries. Our numerical prediction of permeability will be compared with results obtained using relations from previously cited literature [4-6].

For permeability computation, we will use Darcy equation (Eqn. 4) to write:

$$K = \eta\Phi \frac{\sum_i \int_{\Omega i} (1 - I_s) v d\Omega_i}{\sum_i \int_{\Omega i} (1 - I_s) \nabla p d\Omega_i} \tag{5}$$

where the subscript i is for the mesh elements, and Φ is the porosity.

NUMERICAL RESULTS

For our 2D simulations, we impose zero velocity on fibres. Then, we impose a pressure gradient to produce flow, and symmetric boundary conditions on other faces of the REV. First results show that results are not depending on REV size for a periodic packing. The use of anisotropic adaptive mesh on interface gives accurate results without expensive computational costs with respect to an isotropic mesh.

In the following, we plot the variation of permeability as a function of fibres volume fraction. Numerical results obtained with the monolithic approach, are compared with analytical law from [4-6]. Results are given on Fig.3. It can be seen first that permeability increases when fibres fraction decreases. We can see that our results are in good agreement with literature. Lubrication models give better results for high fibre volume fraction whereas cell models are valid for low fibres volume fraction. We can see that our results are close to lubrication models for high fibres volume fraction and close to cell model at low fibres volume fraction.

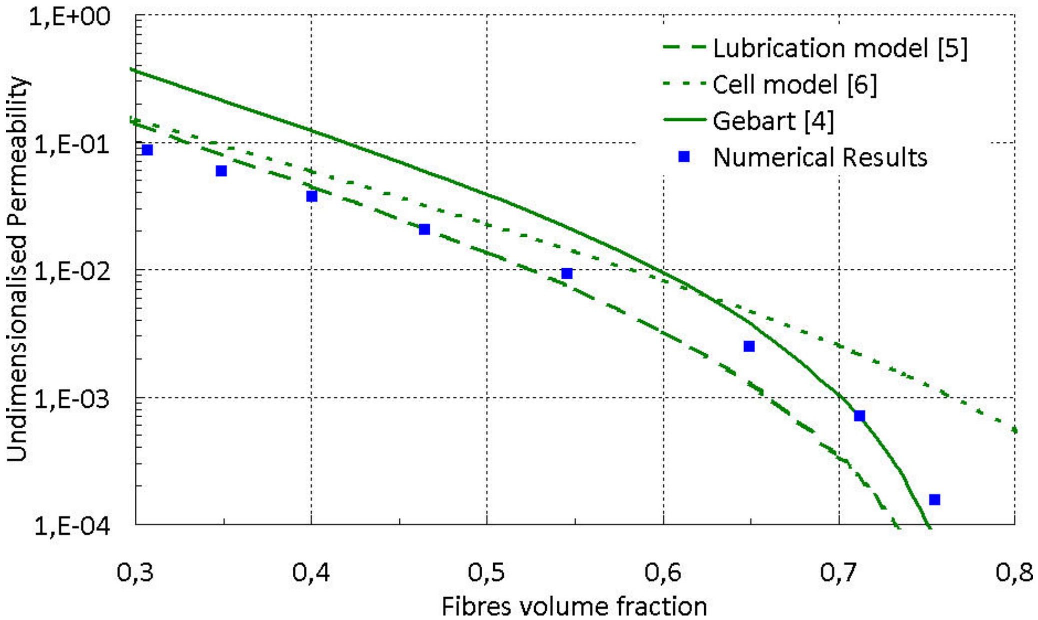


Fig. 3 Comparison of the numerical results with those provided by the literature.

CONCLUSION

Permeability calculations are made using finite element method and are in good agreement with literature. The particularity of the monolithic approach is the resolution of the same equation for all parts of the multiphase problem. This method, coupled with an efficient remeshing tool, allows simulations of complex geometries like woven reinforcement (for the mesoscopic scale). It is possible to take into account for deformation of the solid and front propagation for multifluid problems.

The next step of this study is to add a third phase (air) in the problem. By adding surface tension forces, it will be possible to estimate a non-saturated permeability. Indeed, non-saturated permeability is an important domain of study in liquid composite molding processes [7].

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