COMBINING A LEVEL SET METHOD AND A STABILIZED MIXED FORMULATION P1/P1 FOR COUPLING STOKES-DARCY FLOWS: APPLICATION TO THE RESIN INFUSION-BASED PROCESSES.

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ABSTRACT: The aim of this work is to focus on the Stokes-Darcy coupled problem in order to simulate numerically, with the finite element method, composite manufacturing processes based on liquid resin infusion. In this study, a macroscopic description is used. The Stokes and Darcy are coupled and solved by a mixed finite element method. The originality of our approach is to consider one single unstructured mesh. The level-set framework is used to represent the interface between Stokes and Darcy and also to capture the resin flow front. Examples of numerical simulations are presented which intend both to validate the approach and propose some simulations of such manufacturing processes by resin injection/infusion.

KEYWORDS: Infusion, Stokes-Darcy coupled problem, Beaver-Joseph-Saffman condition, Hughes Variational Multiscale, Level Set.

INTRODUCTION

Resin infusion-based processes are efficient manufacturing techniques to elaborate composite structures with organic matrix. They consist in infusing liquid resin through the thickness of the reinforcements. This apparent simplicity can turn into major difficulties when tuning manufacturing parameters to get proper final dimensions and fibre volume fractions. Numerical modelling of such processes have become mandatory to understand, and then optimize, these processes.

Infusion processes can be modeled as follows: a mold contains initially some liquid resin (a purely fluid domain) and preforms (a porous medium). Under the effect of a mechanical pressure applied on the whole stacking, the resin flows into the preforms, which are themselves subject to large deformations. This paper focuses on the Stokes-Darcy coupled problem in order to simulate numerically the composite manufacturing processes by resin infusion using the finite element method. In this study, for computational efficiency, a macroscopic description is used.

STOKES-DARCY FLOWS REPRESENTATION

Stokes-Darcy coupled problem

The computational domain can be divided into two parts: a purely fluid domain Ω_s representing the liquid resin and a porous medium Ω_d standing for the fibrous network of the preforms (Fig. 1-a). Γ is the interface separating and coupling these two non-miscible sub-domains. It is postulated that the fluid flows in the purely fluid domain according to the Stokes equations (Eqns 1), while inside the preforms the fluid flows according to the Darcy equations (Eqns 2) :

$$\begin{aligned} -div(2\eta\,\mathring{\varepsilon}(\mathbf{v}_s)) + \nabla p_s &= 0 \text{ in } \Omega_s \\ div(\mathbf{v}_s) &= 0 \text{ in } \Omega_s \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\eta}{K}\mathbf{v}_d + \nabla p_d &= 0 \text{ in } \Omega_d \\ div(\mathbf{v}_d) &= 0 \text{ in } \Omega_d \end{aligned} \quad (2)$$

where η is the fluid viscosity, $\mathring{\varepsilon}$ is the strain rate tensor, K is the fibre network permeability taken as isotropic here, and $p_{(s,d)}$ and $v_{(s,d)}$ stand for pressures and velocities in Stokes and Darcy's domains respectively.

In complement, interface conditions have to be considered on Γ. These conditions are the continuity of the normal velocity (Eqn 3-a) due to the mass conservation, the continuity of the normal stress (Eqn 3-b), and the Beaver-Joseph-Saffman condition (Eqn 3-c).

$$\mathbf{v}_d.\mathbf{n} = \mathbf{v}_s.\mathbf{n} \text{ on } \Gamma \quad (3a)$$

$$p_s - 2\eta\,\mathbf{n}.\mathring{\varepsilon}(\mathbf{v}_s).\mathbf{n} = p_d \text{ on } \Gamma \quad (3b)$$

$$2\,\mathbf{n}.\mathring{\varepsilon}(\mathbf{v}_s).\tau_j = \frac{-\alpha}{\sqrt{K}}\mathbf{v}_s.\tau_j \text{ on } \Gamma \quad (3c)$$

where τ_j are vectors tangent to the interface ($\mathbf{n}.\tau_j=0$, j=1,2), α is the slip coefficient, and **n** is the unit vector normal to the boundary of Ω_s.

In order to solve the Stokes-Darcy coupled problem by a finite element method, the weak formulation has to be established in proper functional spaces. It is established by considering both velocity $\mathbf{v}$ and pressure fields p on Ω such that $\mathbf{v}|_{\Omega i} = \mathbf{v}_i$ and $p|_{\Omega i} = p_i$ with i=(s,d). These two fields are solutions of a velocity-pressure mixed formulation defined throughout the whole domain, obtained by summing up the variational forms of the Stokes and Darcy equations [1].

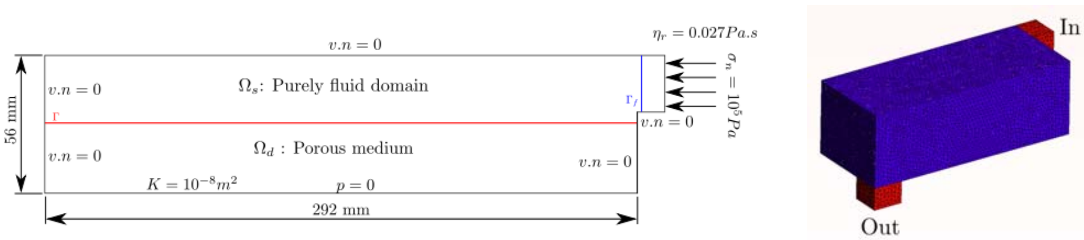
Discretization and interfaces representation

The whole domain is discretized with one single unstructured mesh, made up of triangles in 2D and tetrahedrons in 3D. In the purely fluid domain, P1+/P1 mixed finite element is used. However, this element does not satisfy the Ladysenskaya-Brezzi-Babuska stability condition for the dual formulation used for Darcy's equations. To overcome this difficulty a finite element P1/P1 stabilized with the Hughes Variational Multiscale (HVM) formulation is used to solve the Darcy equations [1,2].

Finally, the Level set method is used to represent the interface between the purely fluid domain and the porous medium. The interface is described by the iso-value zero of a signed distance function ϕ.

NUMERICAL SIMULATION OF THE MANUFACTURING PROCESS BY RESIN INFUSION WITH MOVING INTERFACE

The examples proposed here aim at validating our approach while being also representative of the manufacturing of simple pieces in which resin is injected. The 3D case (Fig. 1-b) is an extension of the 2D case (Fig. 1-a) with a vent placed in diagonal with respect to the resin inlet, essentially to illustrate the 3D resin flow. These examples correspond to some cases which can be simulated with the coupled Stokes-Darcy solver only, *i.e.* preforms cannot deform.



(a) (b)

Fig. 1 Geometry and boundary conditions (a) in 2D and (b) in 3D used to validate the Stokes-Darcy coupling.

The specificity of this simulation is the introduction of a moving flow front, separating the region which is wet, *i.e.* already filled with resin, from the dry region. To describe this moving interface, an Eulerian approach is chosen where the computational domain remains fixed while an additional level-set function ϕ_f is introduced to describe the moving interface. Eventually, two level-set functions are involved in our simulation: ϕ separating the purely fluid domain from the porous medium, which does not move, and ϕ_f which is convected accordingly to the resin flow.

RTM-like processes

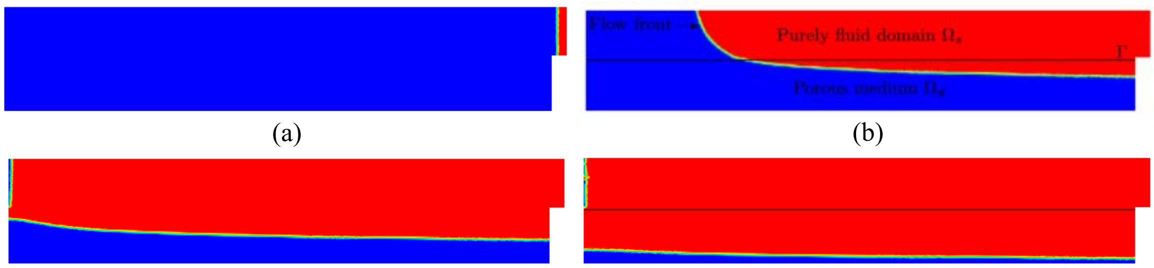
Simulating resin injection in preforms only permits to validate the HVM-based stabilized mixed elements on its own. This simulation corresponds to RTM injection in 3D where the flow front only is used here (level-set ϕ_f). Results presented in Fig. 2 illustrate the fluid flow which is expected from the inlet and vent placements : in the first times resin fills in the geometry, forming an 'elliptic-like' front (Fig. 2-a) which subsequently transforms into a linear fluid front (Fig. 2-b).

(a) (b)

Fig. 2 Flow front (level-set ϕ_f) in 3D injection process (a) → (b).

Infusion/injection-like processes

The case considered now permits to illustrate the Stokes-Darcy coupling (Fig. 1-a). Here both level-set ϕ and ϕ_f are used. The boundary conditions prescribed for this simulation are depicted in this Figure. Fig. 3 shows the change in the flow front during the process, revealing that the infusion in the porous medium occurs before the complete filling of the purely fluid domain (Fig. 3-b). This implies that the flow through the porous medium is not one-dimensional, as may be assumed in simplistic approaches. Moreover, the results also show that when the front reaches the left-hand side of the domain, air is entrapped at this location (Fig. 3-c), causing defects. These results illustrate the ability of these simulations to give hints on the potential defects with can be subsequently reduced by placing properly the vents in an optimization process.



(a) (b)

(c) (d)

Fig. 3 Level-sets position during fluid impregnation in 2D (a) → (d).

The corresponding simulation can be run in 3D, as illustrated in Fig. 4. But in this case, resin will obviously flow easily towards the vent. Again, both interfaces can be precisely described.

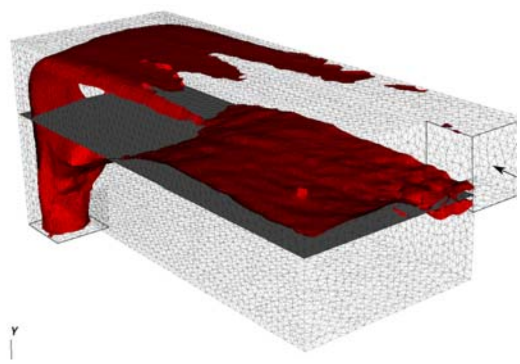


Fig. 4 Example of 3D infusion : level-sets positions during resin impregnation.

CONCLUSIONS

A unified strategy has been developed to solve the Stokes-Darcy coupled problem. A P1+/P1 finite element was used to discretize the purely fluid domain. Since it is known that this element is not stable to solve the Darcy equations, a P1/P1 finite element stabilized with a multi-scale formulation was used in the porous medium. 2D and 3D simulations of manufacturing processes by resin injection/infusion were presented. In these simulations, two signed distance functions were used, one to represent the interface between the purely fluid domain and the porous medium and a second to capture the flow front.

Regarding the on-going and future work, we shall take into account the preform compaction during infusion-driven processes such as in LRI. An updated Lagrangian scheme will be considered relying on displacement-based finite element. A special care will be paid to the interaction of the preform deformation and the resin infusion : resin pressure modifies the wet preform response while permeability preforms depends on the current deformation state [4]. Globally, difficulties that one faces in simulating such processes are then two-fold : accounting for the resin/preform interactions during infusion (Terzaghi's model, deformation-permeability relationship), and representing a very thin resin layer where Stokes-Darcy interface and impervious boundary conditions on either sides yield highly sheared dominated flows. The very first results can be seen in Fig. 5 where the 2D test case (Fig. 1-a) has been submitted to compaction prior to infusion, modifying substantially the resin flow regarding the more basic case in Fig. 3.

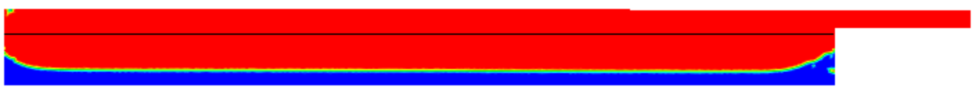


Fig. 5 Infusion of the 2D test case : level-sets positions during resin infusion.

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