NUMERICAL SIMULATION OF RESIN FLOW IN FIBER REINFORCEMENT WITH STOCHASTIC PROPERTY FIELD

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ABSTRACT: In order to account for the influence of randomness in fiber property on the resin flow through fibrous media in mold filling, stochastic simulation method is developed for this physical system, based on the spectral stochastic finite element method. As examples of application, the variability in resin pressure field in different cases of injection strategy is estimated and discussed, providing an idea of randomness propagation in liquid composite molding processes. The current work constitutes the primary step for stochastic simulation of mold injection process.

KEYWORDS: Stochastic simulation, Random field, Spectral Stochastic Finite Element Method, Monte Carlo Simulation

INTRODUCTION

Permeability field in fiber reinforcement often exhibits high spatial and batch-to-batch variation [1], which leads to significant variability in the filling quality of composite parts produced by RTM, for instance. However, the randomness in such process has only been investigated in a few studies [1, 2] in which the Monte Carlo Simulation (MCS) is the only approach. Since the real fiber permeability should be characterized as a random field with both local variability and spatial correlation, the Spectral Stochastic Finite Element Method (SSFEM) [3], which does not rely on repeated deterministic simulations, is more efficient than MCS in the propagation of randomness and estimation of the random flow behavior in mold filling. In this paper, a stochastic simulation method based on the SSFEM is developed for steady resin flow through fibrous media with random permeability field. The formulation and application of this method are briefly explained in Sections 2 and 3, respectively, with conclusions given in Section 4.

STOCHASTIC MODELING OF RESIN FLOW IN FIBROUS MEDIA

In order to introduce the random permeability field $K(x,\omega)$ (x and ω denote the spatial and random variables, respectively) into the PDE system of resin flow, the Karhunen-Loève Expansion (KLE) [3] is applied to approximate $K(x,\omega)$ in terms of a finite number of uncorrelated random variables

$$K(x,\omega) = \overline{K}(x) + \sum_{i=1}^{N_{KL}} \sqrt{\lambda_i}\,\phi_i(x)\xi_i(\omega) \tag{1}$$

using $\{\lambda_i\}$ and $\{\phi_i(\mathbf{x})\}$ ($i=1\sim N_{KL}$), i.e., the N_{KL} highest eigenvalues and eigenfunctions of the covariance of permeability field. The covariance function quantifying both local variability and spatial correlation should be fitted from experimental sample data (for the moment an empirical form is used in Section 3). Substituting Eqn. 1 into the governing equation for resin pressure, and assuming $K(x,\omega)$ to follow a Gaussian distribution so that $\xi_i(\omega)$ ($i=1\sim N_{KL}$) are standard Gaussian random variables, the system of equations ready for applying MCS is obtained

$$\nabla \cdot \left[\left(\overline{K}(x) + \sum_{i=1}^{N_{KL}} K_i(x)\xi_i(\omega)\right) \cdot \nabla p(x,\omega)\right] = Q(x) \tag{2}$$

In comparison to MCS, the SSFEM is able to solve the stochastic PDE system Eqn. 2 in both spatial and random dimensions simultaneously. To achieve this, the solution ($p(x,\omega)$) is represented by the same set of random variables $\xi = \{\xi_i\}$ ($i=1\sim N_{KL}$) as in Eqn. 1, by means of the Polynomial Chaos Expansion (PCE) [3]

$$p(\mathbf{x},\omega) = \sum_{j=0}^{Np} \psi_j(\boldsymbol{\xi})\, p_j(\mathbf{x}) \tag{3}$$

where $\psi_j(\xi)$ ($j=0\sim N_p$) are N_{KL}-dimensional polynomial basis (polynomial chaos) of order up to P. As both the property and solution random fields are respectively discretized in random dimension, Eqn. 2 can be subjected to spatial discretization by FEM formulation and rewritten in the form

$$\sum_{j=0}^{N_P}\left(\sum_{i=1}^{N_{KL}} H_i(K_i)\xi_i\right) p_j \psi_j = f \tag{4}$$

where $K_i = \sqrt{\lambda_i}\phi_i$ ($i=1\sim N_{KL}$) are deterministic KLE coefficients and p_j ($j=0\sim N_P$) the unknown PCE coefficients. By means of the Galerkin method, Eqn. 4 is solved by forcing the residual to be orthogonal to polynomial chaos basis, resulting in a block-sparse system of equations

$$\sum_{j=0}^{N_P}\left(\sum_{i=0}^{N_{KL}} \langle \xi_i \psi_j \psi_k \rangle H_i\right) p_j = f_k \quad (k=0\sim N_P) \tag{5}$$

from which the nodal PCE coefficients p_j ($j=0\sim N_P$) are solved by iterative numerical techniques. Low order statistical moments of pressure field can be obtained directly from the PCE coefficients, and its probability distribution can be estimated from MCS of Eqn. 3 with minor computation effort. Similarly, the Darcy's velocity field can be expressed using Eqn. 1 and Eqn. 3 as

$$u(x,\omega) = \sum_{i=0}^{N_{KL}} \sum_{j=0}^{N_P} \xi_i \psi_j(\xi) u_{ij}(x) \tag{6}$$

with $u_{ij}(x)$ derived from KLE and PCE coefficients. The statistics of Darcy's velocity field are also important for modeling moving flow front.

NUMERICAL EXAMPLES

Application of the current method is briefly illustrated by two examples using the same mold model with multiple injection gates and air vents (see Fig. 1). In example 1, gates A1~A4 are used as inlets and B1~B4 are outlets, while example 2 is the reverse case. The effective permeability (K_{eff}) is assumed as a Gaussian random field with mean value 1×10^{-9} m^2 and an exponential covariance function [3] with correlation length 0.25m. Variability in permeability is quantified by its coefficient of variation ($CV(K_{eff})$). The mold has thickness of 5mm and fiber volume fraction of 0.3. Resin (with viscosity of 0.1 Pa·s) is flowing at constant rate of 5×10^{-6} m^3/s (total of all inlets) into the mold.

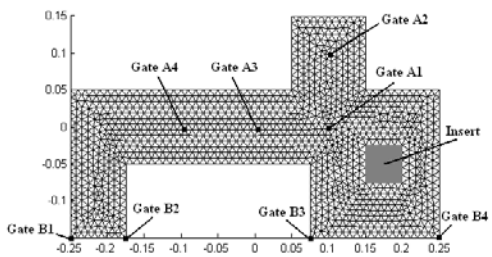


Fig.1 Finite element model

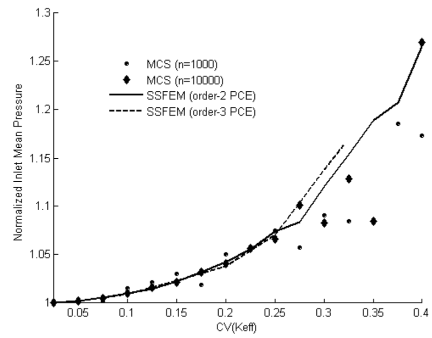
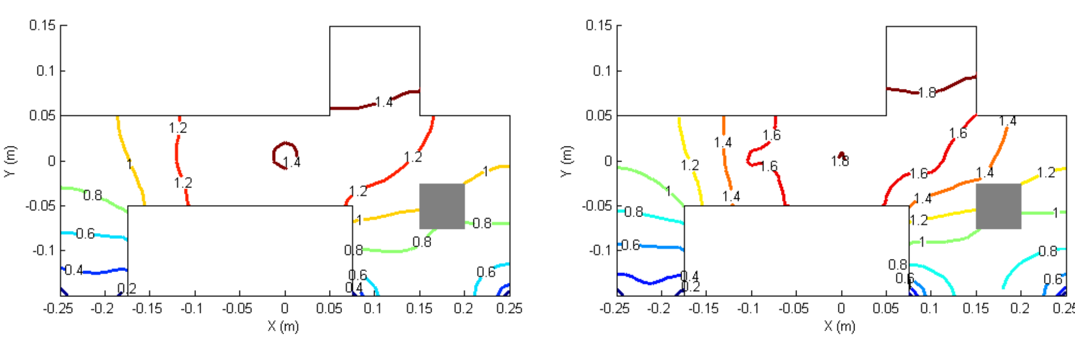


Fig. 2 Normalized mean inlet pressure (inlet A1, example 1)

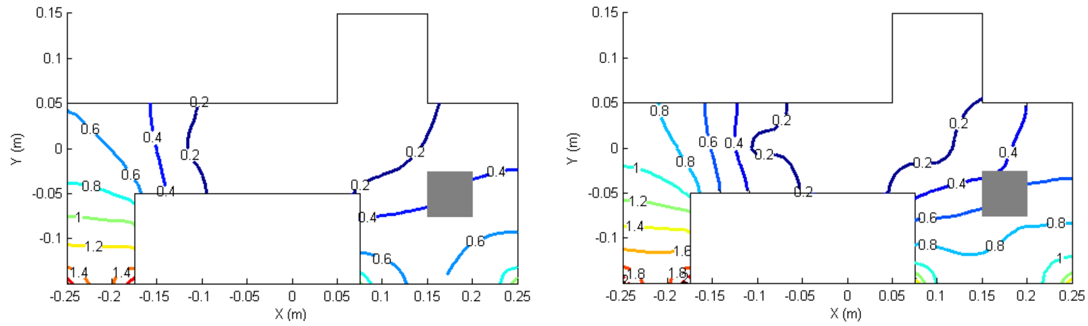
Comparison between Deterministic and Stochastic Pressure Field

Due to the randomness in fiber property, mean pressure at injection gates differs from the value predicted by deterministic simulation, as shown in Fig. 2 (mean values are normalized by the corresponding deterministic value). For $CV(K_{eff})\leq0.25$, results from the SSFEM coincide well with those from MCS (10^5 loops), while advantage in efficiency is obvious for the former which costs about 48 minutes (order-2 PCE, Intel Xeon X5550/2.67GHz) in comparison to 9 hours for MCS (10^5 loops). This advantage will be significant for stochastic simulation of moving flow front. On the other hand, MCS may be unable to produce converged results for relatively high variability (e.g. for $CV(K_{eff})\geq0.3$, 10^5 loops still cannot converge), while the SSFEM is capable for $CV(K_{eff})\leq0.4$. In Fig. 3, distinct differences are observed between the mean pressure distribution ($CV(K_{eff})=0.4$) and the deterministic case, for both examples. Randomness in fiber permeability results in the enhancement of the expectation of pressure magnitude all over the mold, most greatly in the neighborhoods of the injection gates.



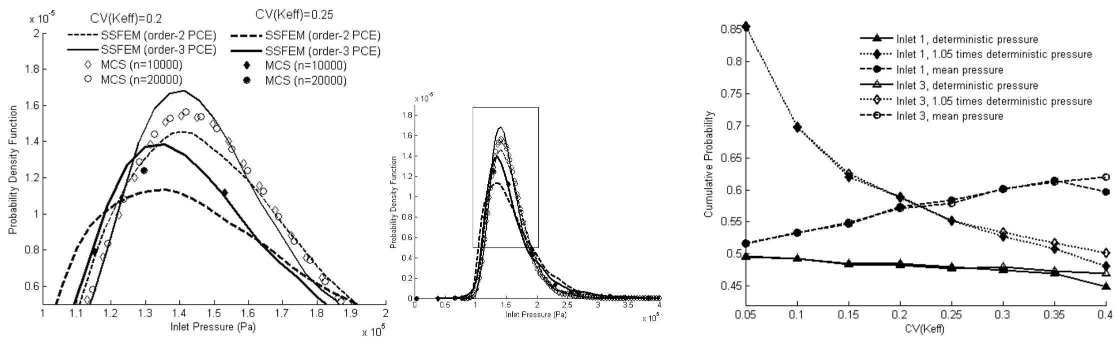
(a) Deterministic solution for example 1

(b) Mean value of stochastic solution for example 1



(c) Deterministic solution for example 2 (d) Mean value of stochastic solution for example 2

Fig. 3 Pressure distribution from deterministic and stochastic simulations (unit: 10^5 Pa)



(a) Probability density function (left: near peak; right: overall shape) (inlet A1, example 1)

(b) Cumulative probability ~ $CV(K_{eff})$ (inlets A1, A3, example 1)

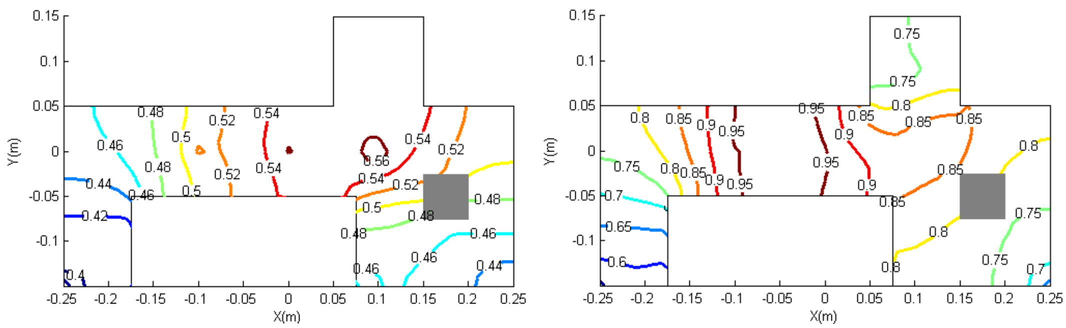
Fig. 4 Probability estimation of inlet pressure

Local Variability in Pressure Field

Since the response field is a random variable at any spatial point, its local variability, quantified by standard deviation, probability density function (PDF), cumulative density function (CDF), etc., is important for reliability analysis. Fig. 4(a) shows the estimated PDF for inlet pressure for $CV(K_{eff})$=0.2 and 0.25, respectively. Comparing to MCS (10^4 and 2×10^4 loops), the current method predicts the overall PDF accurately and captures the peak region better, especially for relatively high material variability (MCS with 5×10^4 loops cannot give correct PDF for $CV(K_{eff})$=0.25). Order-3 PCE predicts the peak more accurately, but order-2 PCE can be applied in wider range of variability (up to $CV(K_{eff})$=0.4), which is necessary for realistic fibrous material with high scatter level. In application, the CDF (integrated from PDF) is more convenient for estimating the probability for a given range of magnitude. For example, given $CV(K_{eff})$ varying from 0.05 to 0.4, the probability for inlet pressure (inlet A1 in example 1) not exceeding the deterministic value varies from about 0.5 to 0.45, as shown in Fig. 4(b). Other curves give the probabilities for its not exceeding the mean value or 1.05 times the deterministic value, respectively.

For providing an overall idea of the variability in pressure field in the mold, the distribution of *CV* of pressure are displayed in Fig.5 for $CV(K_{eff})$=0.4, for both examples. It can be seen that the variability of inlet pressure is much higher in example 2 than in example 1. When the stochastic simulation is extended from steady flow to moving flow front case, the injection gates are usually opened progressively, and local variability can have significant influence on the filling pattern and other variables.

Therefore, the statistical information obtained from the current method is useful for reliability analysis in the design of mold and injection strategy.



(a) Example 1 (b) Example 2

Fig. 5 Distribution of variability in pressure field

CONCLUSIONS

As the first step of modeling the mold injection process involving randomness in materials and manufacturing conditions, stochastic simulation of steady resin flow in random permeability field is treated in this paper. The current method, based on the SSFEM, provides a more efficient numerical tool than MCS for predicting the expectation and local variability of flow response resulted from random fiber property.

The influence of uncertainty on the pressure field is illustrated by simple examples. These results are important for estimation of the realistic pressure magnitude at the injection gates, in order to determine the correct holding pressure on the mold during injection. Besides, they are useful for designing the mold configuration (e.g., numbers and position of inlets and outlets) in purpose of reducing the influence of material randomness.

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