ALTERATION OF PERMEABILITY CAUSED BY TRANSVERSAL FLOW-INDUCED DEFORMATION OF FIBRES DURING COMPOSITES MANUFACTURING

Vilnis Frishfelds, J. Gunnar I. Hellström, and T. Staffan Lundström

Department of Fluid Mechanics, Luleå University of Technology, SE-97187 Luleå, Sweden: vilnis.frishfelds@ltu.se

ABSTRACT: Fluid flow-induced alteration of the permeability regarding deformable systems of fibres is studied. When considering the elastic deformations of the fibre bundles they are founded on the structure of non-crimp fabrics. Transversal flow through random arrays of aligned fibres is considered by using a combined methodology of directly solving the two-dimensional Navier-Stokes equations for the flow in the vicinity of a single fibre and minimisation of the dissipation rate in a system of fibres. The permeability of random arrays of a large number of fibres increases but it can also decrease but then for structured or small systems.

KEYWORDS: flow-induced deformation, permeability, fibre bundles, stitching.

INTRODUCTION

We here model low Reynolds number flow of a viscous fluid through fabrics that have been formed by stitching bundles of fibres together. This flow takes place on two scales within the fibres bundles and between them. Of particular interest is how the fibres and the fibre bundles deform and redistribute changing as well the detailed flow field as the overall permeability. This is especially important in composites manufacturing with liquid moulding processes such as Resin Transfer Moulding and Vacuum Infusion where relatively high pressure gradients drive the fluid flow and the properties of the final composite are crucially dependent on the orientation and distribution of the fibres. The Stokes drag forces generated by the flow results in elastic deformations that are only of importance during processing but may also result in permanent plastic deformations that will affect the final properties.

Since the systems to be modelled consist of a large number of fibres, a combined approach is applied where the system is discretized using modified Voronoi diagrams and the solutions of the Navier-Stokes equations are applied for each part of the division. Then these small parts are combined into a complete system using the fact that the distribution of velocity obeys the principle of minimal dissipation rate of energy, i.e., by minimising the dissipation rate of energy we obtain a linear system of equations with respect to the stream function. Afterwards, the local change of the stream function gives the required stresses for each sector of a fibre and thus the total force and momentum given to a fibre leading to redistribution of the fibre structure in the medium [1]. In order to properly study statistical effects of permeability alterations to the system should consist of a large number of unequal fibres with an arbitrary distribution because the change in the overall permeability is a many body effect [2]. Solving the flow in such a system directly becomes too computational heavy. Berlyand and Panchenko [3] proposed a discrete methodology aimed at finding the effective viscosity of a two-dimensional random array of equal sized fibres using Delaunay triangulation and minimising dissipation rate afterwards. Our approach is in many aspects similar with the major differences that the local analysis of the flow through single gaps between neighbouring fibres is treated differently and focus is set on the deformation of fibres.

DISCRETIZATION AND MINIMISATION OF DISSIPATION RATE

Two-dimensional system of fibres is considered. Assuming that the fibres themselves are impermeable, the stream function for the surface of each fibre is constant $\psi=\psi_i$, where $i=1\ldots n$ is the index of the fibre. The difference in the stream function between any two fibres is just the flow rate in the gap between the two fibres in question. In order to derive this distribution the system is divided into n parts, so that each part contains one fibre. We use a modified version of the Voronoi diagrams consisting of straight lines also for fibres with unequal size [1] for that purpose. The value of the stream function and the vorticity $\boldsymbol{\omega} = \nabla \times (\nabla \times \boldsymbol{\psi})$ between fibres i and j are ψ_{ij0} and ω_{ij0}, respectively, at the crossing with the Voronoi lines (see Fig. 1). The quadratic average of vorticity in the area S_{ij} at fibre i adjacent to fibre j is

$$\left\langle \omega_{ij}^2 \right\rangle = \left\langle \tilde{\omega}_{ij}^2 \right\rangle + 2\omega_{ij0}\left\langle \omega_{ij} \right\rangle + \omega_{ij0}^2 , \qquad \left\langle \tilde{\omega}_{ij}^2 \right\rangle = A_{ij}\frac{(\psi_{ij0}-\psi_i)^2}{d_{ij0}^4} , \ \left\langle \omega_{ij} \right\rangle = C_{ij}\frac{\psi_{ij0}-\psi_i}{d_{ij0}^2} , \quad (1)$$

where A_{ij}, C_{ij} emanates from the average vorticity in a small area S_{ij}; d_{ij0} is the distance between fibre i and the Voronoi line that separates fibres i and j; ~ denotes the case of equal sized fibres i and j. The total dissipation rate of energy approaches a minimum, [3]. So, the following integral is calculated over the total area should be minimised: $\Phi[\psi] = \frac{1}{2}\mu\int \omega^2 dS$, μ is viscosity. The integration can be discretized with use of Eqn. 1 over all of the triangles. Afterwards, the total sum must be minimized with respect to the stream functions ψ_i and middle values ψ_{ij0}, ω_{ij0}. The obtained system of equations can be solved by the use of traditional methods for sparse linear systems of equations.

The total force on the fibre can be expressed by means of vorticity near the fibre by accounting for the viscous and normal forces according to:

$$\mathbf{f}_i = \mu r_i \sum_j \boldsymbol{\tau}_{ij} B_{ij} \frac{\psi_{ij0}-\psi_{ij}}{d_{ij0}^2}\Delta\varphi_{ij} , \qquad B_{ij} = \frac{d_{ij0}^2}{\psi_{ij0}-\psi_{ij}}\left(\left\langle \omega_{ij}^{arc} \right\rangle - r_i \left\langle \left(\frac{\partial \omega}{\partial n} \right)_{ij}^{arc} \right\rangle \right), \quad (2)$$

where B_{ij} is a dimensionless variable characterising the ratio between the average vorticity along the arc at the border of the fibres and difference of the stream function at the positions indicated, see Fig. 1b. The sum of all drag forces equals the driving pressure difference if wall effects are neglected. Hence, the permeability **K** follows from Darcy law: $\langle \mathbf{v} \rangle = \mathbf{K} \sum_i \mathbf{f}_i \Big/ \left\lfloor \mu \sum_{ij} S_{ij} \right\rfloor$.

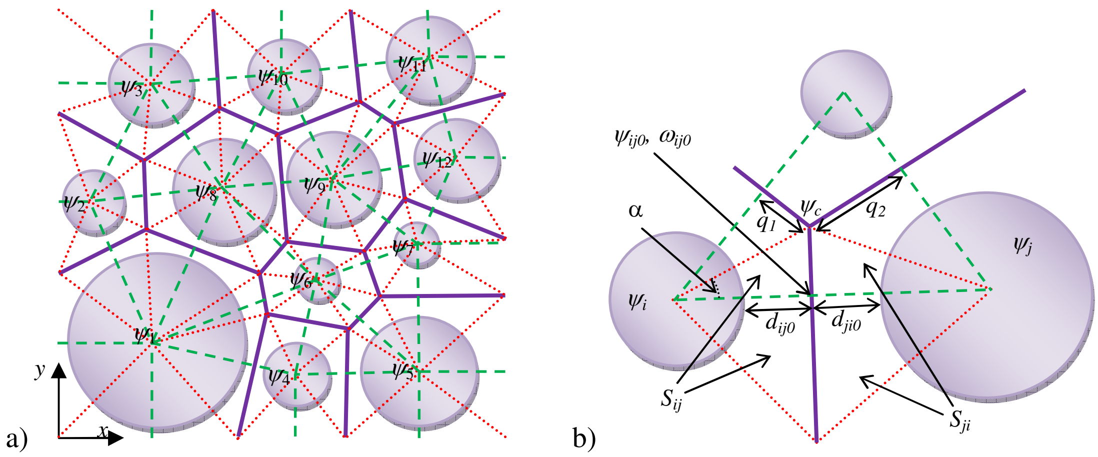

Fig. 1 a –Modified Voronoi diagrams (solid), Delaunay triangles (dashed), and small triangles (dotted). b – Voronoi diagram between three particles.

The dimensionless variables *A*, *B*, *C* Eqn. 1-2 are obtained from simulations with Computational Fluid Dynamics (CFD) performed with ANSYS CFX 11.0. The simulations are carried out with boundary conditions representing a well structured repeatable material with varying geometry [1]. The obtained values for *A*, *B*, *C* slightly differ for rectangular and hexagonal packings. Therefore, the system is analysed locally to see which packing is most representative, see Fig. 1.

ELASTIC DEFORMATIONS OF FIBRE BUNDLES

For composites that consist of fibre bundles, the flow-induced motion of fibres is restricted mostly by elastic deformations of the fibre networks. Let us consider the fibres as long trees in dense forest. The root of the tree is attached to the ground whereas the upper part of the tree shifts in the strong wind. The elastic force that opposes the wind force is just Hooke's law: $\mathbf{f}_i^R = -k_i(\mathbf{R}_i' - \mathbf{R}_i)$, where $\mathbf{R}_i$ and $\mathbf{R}_i'$ represents the new and original position of tree *i*, respectively. In order to account that slender trees bend easier, the spring constant is set proportional to the cross-sectional area of the tree: $k_i = \pi r_i^2 k_0$. We can define a dimensionless pressure gradient **u**, i.e., the pressure gradient with respect to the elastic force when the fibres are shifted by a distance of $\bar{r}_0$ in the flow direction resulting in substantial change of permeability: $\mathbf{u} = (\nabla P)_0 \big/ \left[k_0 \bar{r}_0 (1 - \Pi)\right]$.

The exact calculation of deformation of fibre bundles is rather complicated. Therefore, the system is significantly simplified assuming that the bundles are stitched together with distance l_0 between the stitches. For a perpendicular flow there are three important cases: stretched fibres, non-stretched thin or thick fibres. According to [4], the mechanism for non-stretched thick fibres dominates for fibres with radius above 10 μm. The corresponding solution for the transversal disposition *y* of the fibre is: $y(z) = f z^2 (z - l_0)^2 \big/ \left(6\pi r^4 E\right)$, where *E* is Young modulus, *f* – flow-induced linear force density. The average shift has a linear dependency on the force leading to the following expression for non-dimensional pressure gradient

$$\langle y \rangle = \frac{1}{l_0}\int_0^{l_0} y(z)dz = \frac{f\,l_0^4}{180\pi r^4 E} \qquad \Rightarrow \qquad \mathbf{u} = -\frac{1}{180\pi}\frac{l_0^4}{r^3}\frac{\nabla P}{E(1-\Pi)}. \qquad (3)$$

If l_0 = 0.01 m, r = 10 μm, $|\nabla P|$ = 10^5 Pa/m, E = 10^{11} Pa, then the non-dimensional pressure gradient becomes u = 0.025. At such a u the influence of flow induced change of permeability is about 5 %.

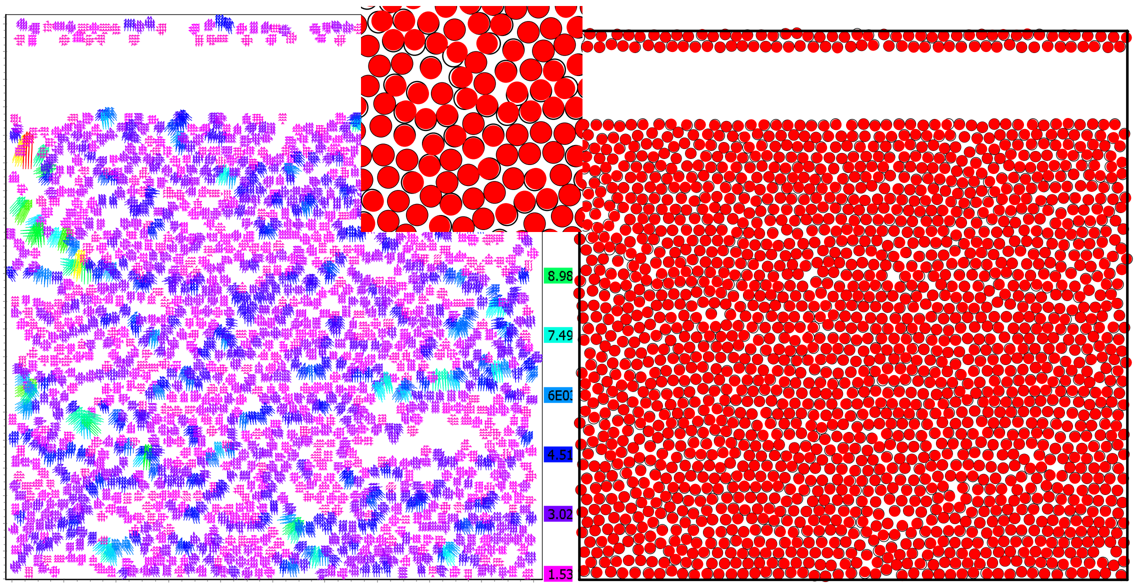

Fig. 2 Left: flow-induced forces on fibres for the flow directed upwards in the figure. Right: redistribution of fibre positions. Hollow circles – original position, filled circles – deformed position. Centre-top: zoomed-in fragment of the right figure.

RESULTS

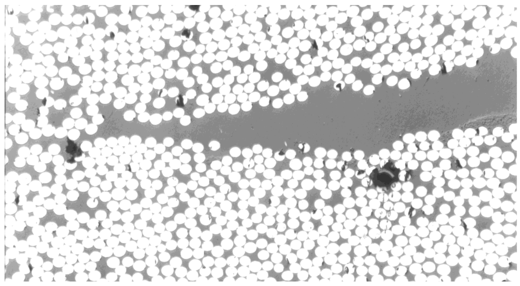

Fig. 3 Experimental cross-section of fibres in bundles

A fully periodic box is used to avoid wall effects. This implies that the total porosity is fixed unless the size of the system is changed. The orientation of the main stream can be arbitrary. The initial positions of the fibres in the system are introduced randomly. Metropolis algorithm with simulated annealing technique is applied to study the motion of fibres where the jump probability depends on the change of energy with this jump. The stochastic distribution of fibres does not only influence the permeability but also result in that the forces $\mathbf{f}_i$ on the individual fibres are generally not directed along the main stream and differ in strength, Fig. 2 left. It is interesting to check the deformations near the bundle boundaries, Fig. 3, but the calculations revealed no significant alteration on boundaries by the parallel flow, Fig. 2 right. Therefore, the analysis of the bulk of the bundle is sufficient for bundles with 1000 or more fibres. Despite the relative shift of fibres is small, Fig. 2 right and centre-top, the change in permeability is essential. These fluid-induced deformations in randomly packaged system result in an almost linear increase of permeability with the flow rate up to u=0.1 meaning that $\Delta K/K_0 \approx \beta u$ (see Fig. 4 left) with a positive constant $\beta \approx 2 \pm 1$ for porosity ranging from 0.25 to 0.35 [1]. Moreover, the role of gap as in Fig. 2-3 is minimal. If u = 0.025, then increase of permeability is ~ 5 %. But the change becomes much higher for higher pressure gradients, thinner fibres or more compliant or weaker stitching within the fabric. Permeability is lower for more compact systems (see Fig. 4 right) that can lead to more rapid alteration of permeability with flow-induced deformations.

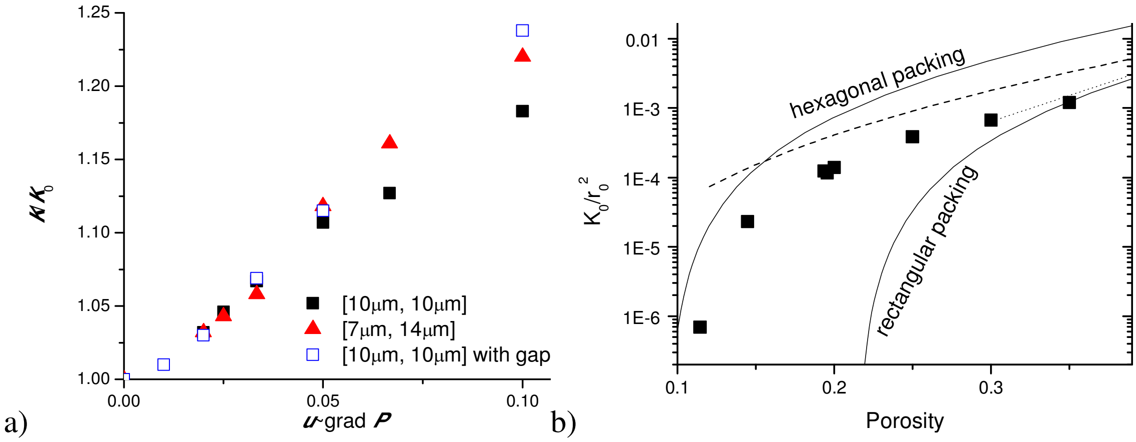

Fig. 4 a) Increase of permeability with flow-induced deformation for equal sized with and without gap and non-equal sized fibres for a random system of ~2000 fibres. b) non-deformed transversal permeability vs. porosity [1]: squares – obtained results, solid – Gebart 1992, dashed – Westhuizen&Plesis 1996, dotted – Sangani&Yao 1988.

CONCLUSIONS

CFD simulations of unit cells of porous media combined with minimisation of dissipation rate of energy of a large system is found to be an effective tool to study flow-induced statistical variations in permeability through randomly packed systems – namely fibre bundles. The permeability of large random arrays increases especially for compact systems with equal size of the fibres. The increase of bundle permeability becomes important for pressure gradients exceeding 10^5 Pa/m. The influence of inflow and outflow regions is negligible in overall change of porosity for parallel flow.

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