EFFECTIVE VISCOSITY IN STOKES-BRINKMAN COUPLING USING THE NAVIER-SLIP FOR TRANSVERSE FLOWS IN DUAL-SCALE FIBROUS POROUS MEDIA

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Introduction

Viscous incompressible flows through and/or over the porous media can be found widely in nature and in industries. For example, the interacting surface flow with the groundwater flow to predict contaminant transport or flooding [1], flow of resins in between the porous fibre tows or fibre bundles in liquid composites moulding (LCM) [2]. In case of slow flows over small scale porous architectures like in the LCM process, the combination of Stoke's equation in the fluid domain and the Brinkman equation in the porous media, denoted by the Stokes-Brinkman coupling, has been considered appropriate in analyses and numerical simulations of dual-scale flows through and over the fibrous porous media, satisfying slip velocity with flows in the boundary layer, once implemented correctly.

In this work, effective viscosity in the Stokes-Brinkman model with continuous interfacial stress has been characterized accurately for the first time and it has been applied to solve various dual-scale flows in fibrous porous media. The effective viscosity is determined such that the interfacial slip velocity can be identified to that in the Navier-slip description and therefore it facilitates the accurate prediction not only of the slip velocity as well as the velocity gradient (stress) at the interface, but also the velocity fields in both porous media (Darcy velocity) and a pure fluid regime.

Characterization of the effective viscosity

Fig. 1 shows a transverse flow over fibrous porous media in the *xy* plane. The upper half of the domain $(0<y<H)$ is the fluid region, denoted by Ω_f and the lower half $(y<0)$ is the porous region, denoted by Ω_p. The pressure difference (Δp) is assigned in the *x* direction. Flow in the fluid region is described by the Stokes equation:

$$\mu \frac{\partial^2 u_f}{\partial y^2} = \frac{dp}{dx}. \tag{1}$$

As for the flow in the porous media, it can be described by the Brinkman equation:

$$\mu_e \frac{\partial^2 u_p}{\partial y^2} - \frac{\mu}{K} u_p = \frac{dp}{dx}, \tag{2}$$

where the subscripts '*f*' and '*p*' represent the fluid and the porous media, respectively. *K* is the permeability of the porous media. The effective viscosity is denoted by μ_e to distinguish it from the fluid viscosity μ. Considering the no-slip boundary condition for the top wall, the velocity and stress continuity on the interface, and in the porous media, the velocity in the porous media reduces from the slip velocity to the Darcy velocity, one can obtain an analytic solution in both fluid and porous media. On the other hand, according to our previous work [3,4], using the Navier-slip model, the solution in the channel can also be derived as a function of the slip coefficient (or slip length). By comparing these two solutions, the relationship between the slip coefficient and the effective viscosity can be derived such that $\mu_e/\mu = \alpha_{BJ}{}^2$. Since the slip coefficient α_{BJ} has been accurately identified in the previous work [3,4], one can readily obtain the value of the optimal effective viscosity.

Verification of the Stokes-Brinkman model with the optimal effective viscosity

Plotted in Fig. 2 is the velocity profile along the line ‘l’ in Fig. 1 for a quadrature packing structure, passing through the centreline of the periodic unit domain, from the direct simulation and the analytic solution of the Stokes-Brinkman equation with the permeability and the optimal effective viscosity, which is computed based on the corresponding actual micro architectures of the fibres. The fibre volume fraction for this case is 0.1. The analytic solution with an identical effective viscosity ratio $\left(\mu_e/\mu=1\right)$ is also plotted for comparison. This line ‘l’ starts from the fluid region, passes through the interface and ends in the fibrous porous media, which belongs to the Darcy region.

It shows that results from the analytic solution with the optimal effective viscosity ratio agree well with the direct simulation, not only for the flow in the fluid region, but also the flow in the porous media with the slip velocity. Note that it has exact the same values of the magnitude of the slip velocity and its slope at the interface as those from the direct simulation. Even in the very thin boundary layer, in which the flow is extremely difficult to be accurately characterized because of the large velocity gradient (the slip velocity is several orders larger than the Darcy velocity), the solution with the optimal effective viscosity ratio still has a good behaviour. The solution with the effective viscosity ratio being one, which is often employed in literatures [5,6], fails to describe such coupling flow especially the flow near the interface: for example, at the interface, the slip velocity from the analytical solution with the identity viscosity ratio is around 3 times larger than the one from the direct simulation. As a key parameter in such fluid flow coupled with the porous media, the slip velocity should be characterized accurately.

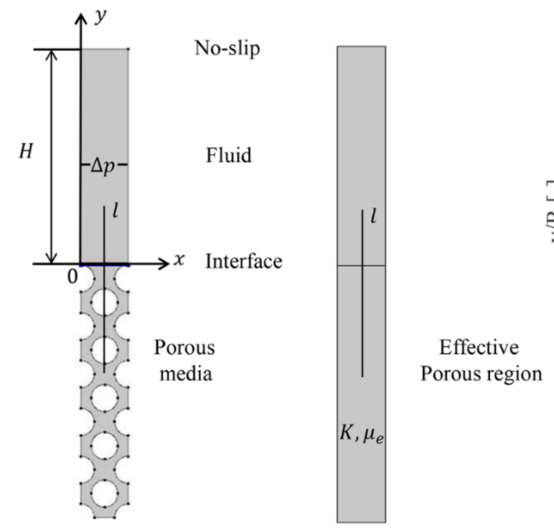

Figure 1: *A schematic description of 1D transverse flow*

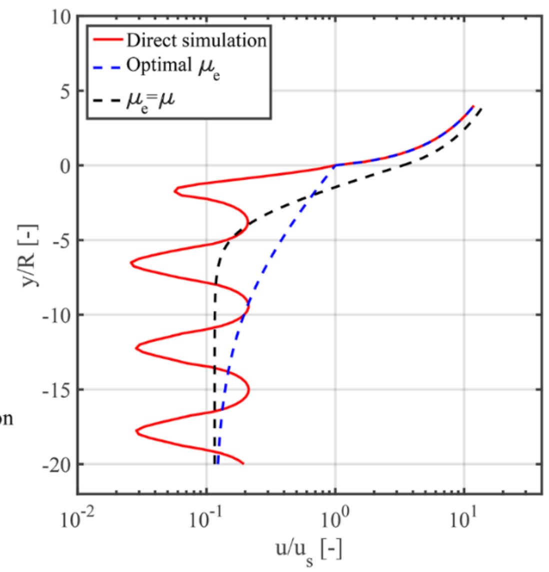

Figure 2: *Velocity profile along line ‘l’*

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References

[1] J. Bear, A. Verruijt. Modelling Groundwater Flow and Pollution. : Springer Science & Business Media, 2012

[2] D. Niedziela, J. Tröltzsch, A. Latz, L. Kroll. On the numerical simulation of injection moulding processes with integrated textile fiber reinforcements. Journal of Thermoplastic Composite Materials. 26(1):74-90, 2013

[3] J.G. Lu, H.K. Jang, S.B. Lee, W.R. Hwang. Characterization on the anisotropic slip for flows over unidirectional fibrous porous media for advanced composites manufacturing, Composites Part A, 100(3), p.9-19, 2017

[4] J.G. Lu, H.K. Jang, W.R. Hwang. Data on the anisotropic interfacial slip length over fibrous porous media," Data in Brief, 13, August 2017, p. 453-459, 2017

[5] W.R. Hwang, S.G. Advani. Numerical simulations of Stokes–Brinkman equations for permeability prediction of dual scale fibrous porous media. Physics of Fluids. 22(11), 113101, 2010

[6] A. Tamayol, A. Khosla, B. Gray, M. Bahrami. Creeping flow through ordered arrays of micro-cylinders embedded in a rectangular minichannel. International Journal of Heat and Mass Transfer. 55(15):3900-3908, 2012