Stokes capillary driven flows in fibrous media

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Introduction

Numerical simulation is a crucial asset for high performance composite manufacturing. During impregnation of carbon fibres reinforcements by a liquid resin, voids may be generated and affect the final mechanical properties of parts. Those voids appear due to the competition between capillary and viscous effects. This study is funded by an industrial chair between Hexcel corporation and Mines Saint-Etienne aiming at modelling and optimising LCM processes for new generation of reinforcements for structural composites.

Mathematical and numerical settings

The finite elements mathematical setting with linear approximation is suited to describe a bi-fluid problem where the liquid represents the resin and the vapour phase represents the air. Solving the Stokes equation gives the velocity and the pressure fields in the domain (fluid and vapour). The interface between the liquid and the vapour is followed and captured with a *levelset* method, which is convected with the flow velocity. Both methods are stabilised with an Algebraic SubScale method [1] and Streamline Upwind Petrov-Galerkin method respectively in order to ensure the existence and uniqueness of the solution.

Pressure jump and gradient pressure jump arise from the sudden change in physical properties such as density and viscosity on either side of the interface. Therefore an enrichement of the pressure space is mendatory to capture physics of a bi-fluid problem since linear approximation is not sufficient. Three degrees of freedom with discontinious and gradient discontinuous shape functions are added on elements cut by the interface [2]. The asset of the extended finite element method used in this method is that the size of the system remains unchanged since the three added degrees of freedom are condensed prior to assembly.

There are three interfaces, first between the fibres and the resin, second between the fibres and the air, and third between the resin and the air. At each interface the jump of the normal stress gives rise to a capillary force substituted into the weak form of the Stokes equations as a Neumann condition. Modifications for each surface tension force, using the Laplace-Beltrami operator [3] yield a new expression allowing to include the mechanical equilibrium of the triple line into the weak formulation. Subsequently, the contact angle is enforced as a natural condition, without explicitly considering the contact angle of the liquid over the solid phase.

The last point of the method deals with the coupling strategy between the *levelset* problem and Stokes problem. As in any bi-fluid problem, the equations describing the flow depend, in a non-linearly way, on the interface position, and therefore on the level-set function [4]. A staggered coupling between Stokes and levelset method is used in this paper. During a time increment, one Stokes problem with capillarity action and one levelset problem are solved.

Numerical results

The test case is a wicking inside a fibre tow. The resin flow is driven only by capillary action for a transverse impregnation into a parallel arrays of cylinders representing carbon fibres. Therefore, in this specific case a 2D simulation is enough to represent the physics of this bi-fluid flow. Physical parameters used as input of the model are given in Table 1 and Table2.

Table 1: *Surface tension values [5]*

	Solid/Vapour γSV	Solid/Liquid γSL	Liquid/Vapour γLV
Surface tension (10^3 N/m)	60.93	30.03	72.8

Table 2: *Properties values for liqud and vapour phases*

	Vapour	*Liquid*
Viscosity (Pa.s^1)	1.71 x 10^{-5}	1. x 10^{-3}
Density (kg/m^3)	1.2	10^3

As it can be seen in Figure 1, initially the interface between resin and air is placed at the bottom of the domain and is represented by a green curve. Due to capillary action, the resin rises inside the domain and the height is reported in the second plot of the Figure 1. At the beginning of simulation, the resin moves with a higher velocity compared to the velocity at the end. Since no gravity is used during the simulation, velocity is impacted by viscous dissipation. The front flow position height is proportional to square root of time verifying the Washburn law as shown in Figure 2.

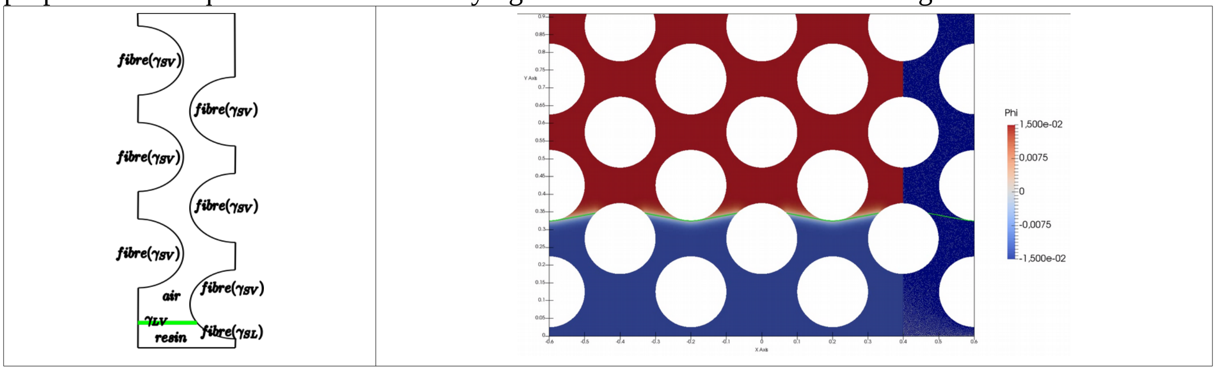

Figure 1: *Boundary conditions and a snapshot during the simulation (axis x and y in cm) – 6 fibres in the domain*

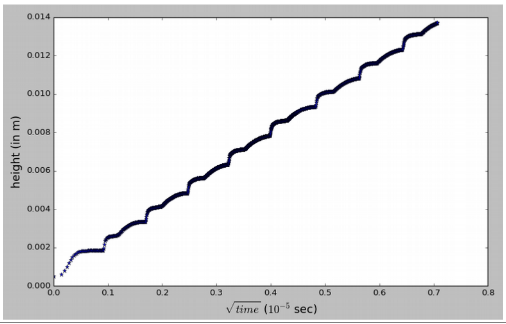

Figure 2: Height of the resin interface as a function of square root of time (9 fibres in the domain).

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